Tone Intervals.

By Jas. Paul White.

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Tone Intervals;

A Series of

12 Monthly Articles

(and one on "The Fret Problem").

Cut from

Jean White's Leader,

of 1891, Boston.

By Ernest Paul White.

Boston: 1892.

With marginal notes and corrections by the author.

James P. White
(36895)
Apr. 25. 1892

Preface.

A preface usually tells us how much better a book might have been had it been possible to make it so! But this book is better than it was possible, under the circumstances, to make it — except by accident — and so the reader may be thankful that the accident took place!

Although I would not be willing to publish this mass of articles in ~~another~~ actual book form, still they contain not a little which I would not willingly ~~let~~ die. Their subject is usually ignored or misunderstood to perfection; and my conceptions of it — so imperfectly here delineated — are different from what I have ever met with.

I have often before, in the past 15 years, tried to write upon this subject; but this series comes the nearest, thus far, to a success.

J. G. W.

Boston, April, 1892.

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TONE INTERVALS.

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Editor of The Leader: —

Do you believe that I can write up, in one newspaper column, either the history of the world, or the life and adventures of a December fly? Well, I can try, as little George said — not, however, to do the above, exactly, but to present something which may substantially aid any thinker on the subject to understand one of the most delightful and useful of sciences: I mean the science of Tone Intervals. I do not mean acoustics,— there are good books on that branch; I mean the arithmetical phase of such intervals; for if there is no such thing as an arithmetical view of them, then the very word *interval*, in music, has no meaning.

Do not tell me, some of you musicians who have dipped into acoustics, or have read works which allude to the subject in hand, that the mathematical or arithmetical view of tone intervals is a dead, unfruitful subject, and amounts to nothing useful in music. There never was a greater untruth ~~uttered~~. Our practical music suffers much from lack of acquaintance with what might, with not great difficulty, be understood by teachers and singers, and the time is coming when no one will dare to profess to be educated in music who is not familiar with musical intervals,— not merely as they are represented in notation or on the keyboard, but as they are in reality, in nature. This is not said in a censorious spirit. I am grumbling at no one, Mr. Editor. There are no text-books which teach this branch (for it really is a branch of science of itself, nearly related to acoustics, but distinct from it), and yet it does exist, some of the time at least, inside of my medium-sized hat; and if any one wants to break the head underneath it for saying this, let him please save the hat which *has* contained so much!

In your November issue I said something about violin tuning and the 612-division of the octave interval, which may be better than nothing as an introduction to a few letters on a hobby — no, I mean a *real horse* — of mine which I have ridden mostly for my own recreation for fifteen years. Dropping the noble quadruped figure of speech, I would like to say, by the way, that I have never for a moment found the subject dull as those do who do not understand it, but it has indeed been my recreation and amusement, although it has been work, work, work, both mental and mechanical. My slowness in giving to the public much of any account of my science and the musical instruments which I have made, has been owing partly to the lack of others' interest in the matter, and partly to my own circumstances, which have kept my work itself impeded the most of the time to an extent which I will not try to describe. Could not some of your correspondents inform me how I can live my life over again in more favorable circumstances and have a little something of the ideas already gained to start with?

I would then start off with some such acquired truths as will appear in the remainder of the present letter and in any that may follow; and here is a little table which any one will be able to *get around* at least, and it will be useful to glance at, once or twice, in reading my letter: —

TABLE OF MUSICAL INTERVALS.

(*The whole animal, uncut.*)

Prime intervals.	Their vibrational ratios.
1.	1 : 2
1.5849625+	1 : 3
2.321928+	1 : 5
2.807355—	1 : 7

The above double table forms complete data for easily finding either the *magnitude* or the *vibrational ratio* of any interval which has anything to do with real music, either as an art or as a science. It will be much more convenient to *use*, however, if we abbreviate it, as follows: —

The four working intervals.	Their vib. ratios.
1.	1 : 2
.5849625+ *	2 : 3
.321928+ †	4 : 5
.807355— ‡	4 : 7

It will be sufficient to allude to the decimal measurements as .585,322, and .807, for three figures only will measure them in absolutely acoustic tune, but for calculations it is necessary to extend them as in the table, especially the first two. I have the first decimal actually carried out correctly to thirty figures, which reminds us of the circumference and diameter — *incommensurable* quantities; and so are musical intervals, the exceptions only proving the rule. But, as in the diameter and circumference and in other geometrical dimensions, we can always easily approximate just as near to absolute perfection as any case requires, three decimal figures only being sufficient to measure a musical interval in tune, and six or seven for purposes of calculation.

The first interval in the table, our grand unit of measurement, is the difference between the pitch of the voice of woman and that of the voice of man. The second is that to which a violin is tuned. As for the other two, they are so roughly represented in the even duodecimal scale of the common keyboard, that I will not undertake just now to tell what they are; but most readers will recognize one of them, and a few will either guess or be sure of the other (4 : 7). They are represented on the common keyboard, as nearly as possible, by .333⅓ and .833⅓, or one-third and five-sixths of the octave interval respectively.

There are three arithmetical or numerical ideas connected with musical intervals at the present time, and one of them should be dropped, as much as possible, so far as teaching the nature of these intervals is concerned. First, because three is one too many, and makes confusion in language; and, secondly, because the third one does not belong necessarily to the subject, and is more an impediment than a help in understanding the nature of music. I refer to *the scale*. I beg not to be misunderstood. I am certainly not an

* 1.584,962,500,721,156,181,452,742+
† 2.321,928,094,887,362,348+
‡ 2.807,354,922,057,604,107+
These were found by carefully dividing logs. 3, 5 and 7 by log. 2, using logs. of over 30 places. I use Hutton's Tables, London: 1822 (very reliable). J.P.W.

Space and decimal point

Octave ... 1:2
Fifth ... 2:3
Major Third ... 4:5
Harmonic Seventh 4:7

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enemy to what is called—rather vaguely—the scale, even in the science of intervals, and much less in the art. I only suggest the idea of ignoring it as much as possible till we come to understand where such a ladder came from, what kind of timber it is, what its true place is, and how much authority it has. As things are, it is made the Alpha and the Omega in music, not merely in the art, but in the science too — if such a science may be said to exist. And I know too well the difficulty which lies before me when attempting, in my letter, to keep the scale out of the way, for the very names of all intervals come from it. The scale itself is well enough, and the names one, two, three, four, five, six, seven, and eight are perfectly proper, but, confound them! they don't know their place. They stand there before the gate of the real temple of musical science, holding the keys pretty much; and will not enter themselves nor permit anybody else to enter. Everything is good in its place, and anything under the sun may be very bad and mischievous out of its place.

In understanding musical intervals, let us have just two numerical ideas, the two in the above table, — just two, because that is just what there are. We can manage two without getting confused in ideas and language; but the third one mixes us up. For instance, supposing we say that a certain ~~lovely and soul-refreshing~~ very harmonious interval, when in tune, is .322 of some chosen unit of measurement, and that this interval thus measured by a decimal fraction results from vibrations in the simple proportion of 4 : 5; that is, when the vibrations are actually, say, four hundred and five hundred per second of time, which would give the interval at a pitch somewhere about the middle of the piano or organ keyboard, — and then, supposing that after we have stated these *two* facts, which I think we are all able to keep in mind as distinct ideas, although they are intimately connected, we wind up with the confusing *other* numerical statement that this interval is a third, a major third! which rather jumbles things. It is said that two are a company, but that three are nobody. The numbers which *measure* an interval, the numbers which express its *vibrational* existence, and the number by which human beings have *named* it, make a confusing trio; and it seems as if certain writers — tacitly and unwittingly — avoid this confusion of numbers, and in what a very neat and easy manner, too; namely, by keeping the scale numbers, — oh, yes! — and jumbling the other two into one! And why do they do such a barbarous deed? Simply because they often do not know the difference between numbers which express ratios of vibration and those which express interval measurement; while a little thought would show them that they cannot be the same thing.

Now as there are but four tone intervals in nature which really *make* all music, these can be named, for scientific purposes at least, without reference to any scale or other melodic run, whether of six, eight, or any other number of tones; and thus we can do away mostly with that other and confusing numerical idea, that ladder or scale, so visible and fundamental indeed in our common musical practice, but so bewildering in all attempts to learn systematically and truly the nature of music.

And there are other reasons for thus trying to place the scale away in the background, in teaching tone intervals, whether we mean the major diatonic, the minor diatonic, the chromatic, or the enharmonic, — for each and all of these terms are more or less vague. I thought very differently long ago, and that was when I regarded "the scale" as something quite divine, a mystery indeed, because, for instance, any two successive tones made a discord, while two alternate ones harmonize! It was the scale, without any other idea back of it, which kept me in that mystery and darkness. We have heard of the man who held on to a rope for dear life, a long time, in the total midnight darkness, and did not know that his feet were but a few inches from the ground.

Now be it known henceforth and forever that there are just two numerical or number ideas which naturally belong to the subject of tone intervals, and that neither of these two is the scale idea.

The simplest of all musical intervals is called by the unscientific name of *octave*, as if the number *eight* had anything to do with it! In science, it might appropriately be called the *dual*, not a new word indeed, but now first used in print in this sense (unless some of my notes or correspondence have got into print without my knowledge). For a like reason, the three other prime intervals might be called the *trial*, the *quincal*, and the *septal*. They are independent of each other and very different, not only in the prime numbers of their ratios, but in the very sound of them, so to speak, — for of course it is not the interval which sounds, but the two tones forming it. These four names would serve not only for the four intervals in the table (the short form), but as generic terms also; for since all possible intervals which belong to either the science or the practice of music are characterized by the *highest* prime number in their vibrational ratios, they will all belong to one or another of these four classes or genera. Thus, the ratios 5 : 6 and 80 : 81 make quincal intervals as well as 4 : 5, the highest prime being five in each; while the ratios 6 : 7 and 63 : 64 form septal intervals as well as 4 : 7. As for the measurements of these and all others, they are easily found by subtracting, and sometimes adding, the decimals in the table.

I have said that the four intervals in the table really *make* all possible music. They are what guide the ear in tuning or singing; so that we may say that we never tune nor sing the smaller intervals, they being merely the *differences* between these prime ones, and take care of themselves, because they have — paradoxically — no independent existence.

But in thus limiting the musical vibrational prime numbers above one to four, — namely, two, three, five, and seven, — I have to beg the pardon of the late Mr. Ellis, the translator of Helmholtz, who teaches, in his second and last edition of Helmholtz's work, London, 1885, that seventeen (!) is the characteristic prime in the so-called "diminished seventh" combination.

It was once taught, away back in the ages, by Pythagoras and his followers, that three is the highest prime number in musical ratios. Since then, and even down into our own century, it has been taught that five is the highest prime in such ratios. Slowly! now, slowly! but there is just one more which I believe that about all respectable writers on intervals now more or less recognize, namely, 7; and music, as known in these late ages at least, could not exist without this, although the intervals arising from this prime are very much abused on our instruments, the voice included, and also in teaching.

The vibrational prime 7 forms the most delicate and also the most misunderstood of musical intervals; and between this prime and the next higher prime, 11, which is quite a wide gap, there is no passing over in music. I can recognize and tune an interval whose ratio is 8 : 11, and possibly I could also 8 : 13, but there is no musical harmony nor melody in them. These higher primes doubtless belong to the domain of acoustics, and therefore have something to do with *quality* of tone, but nothing to do with musical intervals proper. This is my statement; and I am sure that no one who has even become half as familiar with this whole ground as I am could disagree with me.

It would only overweight this article, which is already longer than I propose to write you hereafter, to try to present to your readers the method of arriving at the decimal measurements of the four fundamental intervals in the table. Every mathematician either knows, or can know, that I am right in them. The measurements are found from the vibrational ratios by a mathematical process, which, although easy enough in these modern days of logarithms, would not interest nor be very intelligible to more than one out of a thousand readers. It is necessary to calculate and ascertain these four independently of each other, all others being easily found from them by simply subtracting or adding these, which any one can do after a little attention. All the ratios also, even the finest, are similarly found from the four in the table.

In my future and briefer letters I will take up numerous points which naturally grow out of principles herein laid down.

JAS. PAUL WHITE.

TONE INTERVALS.

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II.

IT will be to the advantage of those who take an interest in these articles to preserve each one for reference; for although I shall not be afraid of repeating, on a subject so little understood and so misunderstood, not everything can be stated twice or more which it may be useful to remember or turn to.

The occupation by which I earn my living demands all my time at the present writing, therefore this number will be a light one. But don't despair, genial reader! You will have plenty of the stuff before the year is out, and it won't be all *stuffing* either. We have read much historical disquisition on music, well-written *stuff* too, which makes it all the more palatable for cultivated readers. It is nothing but scales and gamuts and modes, staves and clefs, flats and sharps, time and eternity, — of absence of any natural science. This may be all very well — I am not trying to tear down anything, I am only proposing, no, purposing, to build a little for myself, that is, for my family, your readers. I wish to leave them something really fireproof, whatever it is. I don't want them to live in the same sort of dwellings as their forefathers have occupied for hundreds and hundreds of years, even although time has made them venerable.

Music, which always has so much to do with what makes up human life, is still practically a mystery, as well to the educated as to the uneducated. The scales, the gamut, the flats, the sharps — what do you mean by them? What makes them? Why wasn't your octave interval one-tenth longer, and the steps of your scale very different in length and in number from what they are? Some would answer — I do not mean any of *you*, my family of readers — "Oh! we don't care about such things. Acoustics teaches all this, I suppose; but we learn music as it is taught," which means learning to play or sing according to the approved method of either this one or that one, and then really knowing less of the nature of music than before. But even reading acoustics, Helmholtz and smaller works, does not do the business, does not give you any clear conception of the beautiful structure of music, of which Nature has furnished the principles; and it is for us to dig them out and systematize them in a rational order. Acoustics, although very interesting as a kindred science, does not do this. Nay, at present it even stands somewhat in the way; for it does not recognize any other science as a basis of music but itself, while the real and practical basis of musical science is an unrecognized branch of mathematics; acoustics belonging to natural philosophy, and teaching many things about sound and tone, but leading us far from the knowledge of tone *intervals*.

There is no name for this branch of science. Geometry treats of lines, surfaces, etc. A tone interval is a certain species of quantity, different from an interval of time or one of space. Musical intervals can be all naturally classified, measured, and nicely systematized, like the facts of any other department of human knowledge. And isn't it funny (?) that for one peculiar reason and another this has not been done long ago? As for *my* doing this, although my devotion to the matter for so many years with unique instruments of my own invention and construction has given me views both of principles and their practical bearing, which I do not find to be known to others, still, being so poor a teacher, I can expect to communicate but a part, and perhaps a very small part of what I have gained. But if my dear readers will bear with such style as I can command, I will present features of this maltreated subject of tone intervals each month, which will certainly be a substantial contribution, anyhow, to the unnamed science.

JAS. PAUL WHITE.

Logarithms are the real means of all numbers with which is 3 octaves plus a major third, is an incement unit for musical intervals: 2, 3 are exactly to the octave itself 17 dividing every logarithm by the log. of 2, making the new log. of 2 = 1, and hence log. of 3 = 1.584962 5 etc.

That is, the measure of the ratio (1:3) is not twice as much as the measure of the ratio (1:2) he regards as 1. It's an old fallacy, I see, with the unit ratio (1:10), forming a tone interval. Just know the historical fact that I had invented the method of expressing the two or three years. J.P.W.

TONE INTERVALS.

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III.

(erased by J. P. W.)

~~TONIAMETRY, Harmology, and Acoustics are the sciences which underlie music. There are others, of course, which have something to do with it; but it is not usually necessary to say, for instance, "physiological acoustics," any more than it would be to say "mathematical acoustics," for acoustics naturally implies something of both mathematics and physiology. But toniametry, a branch of mathematics, the science of measuring *tonia*, and harmology, the science of musical harmony (not mere key-board or notation harmony), are primarily the basis of practical musical knowledge, and alone give us conceptions of the structure and rationale of music.~~

In the present article it is attempted to state and emphasize a truth which scarcely seems as yet to have been born. I refer to the fact that a tone interval is a species of *quantity*, and *tonia* will be its name till some one can select a better. Of course, I realize that I place myself in a dreadfully dangerous position by thus putting things, and even amusing myself by suggesting a few very innocent terms for my own use; but I have so completely realized, for fifteen years, that all the attempts of any one to write about these things, in the common, dubious, ambiguous, inaccurate, inappropriate, and unscientific terminology of music, ends in only confusion and failure, that I can do no less than make a stroke or two for a different plan of attack, on the intellectual faculties of the intellectual readers of THE LEADER. Musical terms there certainly are in abundance; but you will not find, even in the four volumes of Grove's Dictionary, a few such terms as are absolutely necessary for treating of music as a science.

The word *tone*, however, as well as some other musical terms, I am thankful for; but it will be used here only in its first and proper musical sense, and not to mean either quality of musical sound, nor an interval between tones. Its primary sense in music is a *musical sound*, and, of course, such a sound must needs have a certain pitch, according to the vibration number, and the pitch is really the tone, without any reference to the kind or quality of tone; although we say, for convenience, a good tone, a loud tone, etc. These latter ideas are out of the line of these articles, and, like many other matters of secondary importance in music, belong to acoustics. This is not a joke. It may be said by some that acoustics includes all the ground which is now being considered by me; but the fact is, that science, at least as heretofore taught, does not touch it, except to its injury, and, therefore, I prefer to regard the measurement and classification of tone intervals as a separate science from, although, of course, a relative of, acoustics.

From the well-known word *tone*, then, we may easily form the word *tonia*, to signify an interval between two tones. It may be asked why the word *interval* itself will not answer all purposes. The answer is, that that is too general a word. We say an interval of time, an interval of space, and an interval of — what? — difference between tones? No, that is awkward; an interval of tonia, thus having a very simple expression of the idea, and not even being obliged to resort to such a compound as tone-space; in fact, if this new term were made quite familiar, we might leave the very word *space* pretty much where it belongs.

Now, what on earth can be more simple to any one than intervals of time, intervals of space, and intervals of tonia? Eh?

And what is to be done with this word in music? Everything; and the word will give due prominence to an idea which now lies hidden, ignored, demoralized, or all three. It is as impossible to discuss ~~musical~~ questions of intonation in the least well-grounded hope of understanding ourselves, without familiarity with this idea, as it is to see without eyes.

What makes tonia? Different rates of vibration. Correct. And how do we measure tonia? for it was just stated that it is a quantity, as truly so as are time and space, and therefore can be measured; or, rather, it can be measured, and therefore it is a quantity; for a quantity is what can be really measured. But how is tonia measured? I confess that I never talked with more than three or four persons in my life who seemed to have any true idea at all as to how it is measured; and among my correspondents in various parts of the English-speaking world, I cannot count many such either. About as far as one's thought (?) ever goes in that line is to imagine that a musical interval is measured "by the number of vibrations," which is very much like saying that the glorious orb of night was made in a cheese factory, — only much more mischievous; for I suppose one's physical and mathematical ideas of the solar system might be very much out of the way, through want of attention to that department of nature, without necessarily blocking up musical conceptions. No, we cannot measure tone intervals by the numbers of vibrations; but we measure them *by measuring their ratios of vibration*, and, to use a very homely expression, that is something which no fool can do. If you do not believe it, just try it.

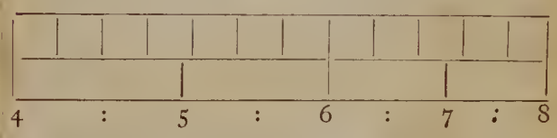
There are, indeed, some cases in which any one might, by a little inspection of the figures of the ratios, ascertain their measurements. Take the ratios 2 : 3 and 4 : 9. We wish now to know how these two ratios compare *as to size;* that is, we wish to measure one by the other. We can pretty easily do it in this particular instance and some others. The second is evidently the larger ratio, and it can be easily separated into two parts; thus, 4 : 6, and 6 : 9, or 4 : 6 : 9. But 4 : 6 simplified, is 2 : 3, and 6 : 9 simplified, is also 2 : 3. So we see that the two parts are equal, and therefore we have measured the second or larger ratio

by the first, and find that it contains just two of them. Other similar cases might be mentioned, — 1 : 4 and 1 : 8 are respectively double and triple the ratio 1 : 2; for 1 : 8 can be separated into three *equal* ratios; thus, 1 : 2 : 4 : 8. Therefore a tone interval vibrating as 1 : 8 is three times one vibrating as 1 : 2; that is, three duals or octaves. The ratio 4 : 5, doubled, is 16 : 25, and tripled, is 64 : 125; but these last two would form discordant tone intervals, the last one, especially, being entirely useless in music; for, speaking after the manner of music, three major thirds do not amount to an octave (as they appear to do on the instrument) by nearly half a semitone, and a more unmusical interval than an octave thus shortened it is difficult or impossible to form. It will be seen from the above figures that *squaring* the ratio numbers *doubles* the interval; raising them to the *third* power triples it, etc. So much, then, for the exceptional cases; that is, those in which it is rather easy to see how many times one interval is contained in another; or, in other words, how they are measured by some one interval chosen for a unit.

But the fact is, such cases as these are not the ones with which we have mostly to deal in music. About every one of the beautiful, harmonious, and melodious intervals — yes, and the so-called "dissonant" ones too — which make up music are not formed from doubling, tripling, etc., nor by cutting any interval into aliquot parts, almost all intervals which make any harmony being indivisible into such parts, so to speak. The musical ratios have different *prime* numbers, 2, 3, 5, and 7, and the four ratios which *make* all music, namely, 1 : 2, 2 : 3, 4 : 5, and 4 : 7, cannot be measured by one another in any such easy way, by mere inspection, as in the exceptional cases aforementioned. But it is above all things necessary to measure all these, if we would truly understand musical science. I do not say it is necessary to understand the whole process by which we arrive at such measurements, for it certainly takes a pretty skilled mathematical mind to do that, — one who could really calculate or *make* logarithms. And this is what I meant by saying that "no fool" could do it, though he, as well as you and I, can *use* these tables of logarithms which mathematicians have made, and which have now, for several hundred years, been of so great use in various departments of mathematical investigation. Neither do I say that it is even necessary to use these tables in order to gain true knowledge of these matters, provided that one has a few concise tables made from them; such, for instance, as I gave in the January article; namely, the relative magnitudes of the four prime intervals whose ratios are above given.

Logarithms are the *measures of ratios*. That is one of the definitions of them; but the unit in the common tables is the ratio of 1 : 10, a very inconvenient one for musical purposes, and so my decimals make the unit 1 : 2,# so that all intervals are measured in decimal fractions of an octave. It is necessary to approximate to six or seven decimal figures for our calculations, and, therefore, the school-book logarithms are not quite accurate enough for this purpose, though the above number of figures gives any interval far, far beyond sensible perfection; for a thousandth of an octave is about the ear's limit, while six figures express millionths. Tone intervals, which are characterized by exact vibration ratios, which we are considering in this article, are usually *incommensurable* quantities, as is so often the case in the higher operations of mathematics; yet we can approximate just as near to mathematical exactitude as any case requires. Logarithms furnish us easy means of finding, in any unit we choose, not only the magnitudes of the incommensurable ratios, but also the commensurable ones, such as I mentioned first, and even measures these easy ones much more easily than by the method I gave. In such concise tables as I would give, the octave, or dual (1 : 2), is one. Then two duals (1 : 4) would, of course, be 2; while the trial (2 : 3), the quincal (4 : 5), and the septal (4 : 7) would be the decimals given in full in Article I., and, abbreviated, are, respectively, .585+, .322—, and .807+.

Now, I shall cause our printer to give here in type a rough approximation of the relative magnitudes of these four intervals which make the whole structure of music; and they are all contained in one five-toned harmony, which, either with or without the last tone (which forms the octave), is called by musicians the chord of the dominant seventh. In juxtaposition is also an octave of the even "chromatic" scale of the key-board: —



The lower ladder cannot be very accurate, of course, in type, but it furnishes a fair idea of the relative sizes of the intervals, and also the amounts by which the same intervals, as tuned in the common scale, vary from true harmony; the quincals being out of tune about one-seventh part of the equal "semitone," and the septals more than twice as much, or about one-third of that "semitone." But that great leading interval, the trial, has the privilege of being almost exactly in tune, in conformity with the system; and, on so small a scale as the width of a newspaper column, its would scarcely appear to the naked eye, and ~~therefore is~~ not attempted here. If the octave were represented by a foot in length, the variation would be about one-third of a sixteenth of an inch, or much more nicely, about one fifty-first part of an inch. If, therefore, in our system, the quincals and the septals were in anything like so nice tune as are the trials, the system would be beautifully harmonious, instead of rather discordant for prolonged tones.

by dividing all common logs. by log 2, .301029995664-.

its very small error

There are some other lights in which I intend to present the idea of tonia, and the fallacy of imagining that it is measured by the "number of vibrations," and I may take it up next time.

I confess, however, that I am not trying to exactly suit everybody in these articles. I do not even suit myself. But I am not writing nonsense, however good and useful that may be at times, especially good nonsense. If I could write out my thoughts on this subject one hundredth part as fast as I can think them, or refer to them, I should get on much faster in these remarks. We have to be so careful in writing and in print, you see, not to say what we don't mean, and to say just what we do mean, that it takes time and too many words at best; and even then we may sometimes trip.

JAS. PAUL WHITE.

TONE INTERVALS.

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IV.

IN the preceding number (March) I dwelt on the idea expressed in the word *tonia*, it being a real mathematical quantity in nature, and deserving a name, although it might have kept on deserving, perhaps, like the rest of us, forever, without getting its deserts, had it not been found useful to mortals to call it by some simple name.

DEFINITION.

Tonia. — The difference between any two tones; a musical interval; a species of quantity, for it can be measured. So we say *an interval of tonia*, as we would an interval of space or of time.

It was also attempted in the last paper to present some idea of measuring tonia, and to show that it is not done "by counting the vibrations," as some thoughtlessly suppose, but by *measuring the ratios of vibration*, which is a very different thing. It was originally a first-rate mathematical task to do this, but it is now made easy in these last centuries by the invention of logarithms. In fact, before the introduction of logarithms (which, however, were not invented for musical science specially, but to facilitate mathematics in general), I think that *Tonia* must have had rather a hard time of it to get justice *meted* out to her, even worse than at present, which is not necessary. That musical intervals result from simple vibration ratios, is an older idea than logarithms; and a still older idea is, that aliquot divisions of a string, illustrated by a movable bridge, as on the monochord, will give the various tone intervals which make music. Pythagoras (born 534 B.C.) and his followers had got as far as that in musical science (although 3 was their highest musical prime number, in theory); but they do not seem to have known of *vibration* ratios; it is only ratios of *string lengths* which they speak of; yet the two sorts of ratios are the same, only one is the inverse of the other. If a musical string, a foot long, between bridges, be nicely stopped by another bridge, eight inches from either end, the part eight inches long will give a tone forming an interval with the tone of the whole string, which I call a *trial* interval, or *trial*. Fiddlers call it a fifth, and so do some few others, perhaps, but I call it the trial — in these articles, if you please — because I here keep within gun-shot of science, you see. I call it trial, not our common word of Saxon origin, but a word formed directly from the Latin *tres*, or *tria* (though the common English word and this happen to be spelled and pronounced alike); and I do so because the prime number *three*, characterizes the ratio of the string lengths, but more particularly the vibration ratio, which makes the tone interval; that ratio being 3 : 2 in the first case and 2 : 3 in the second, just the same, only the order is inverted; shorter string, higher tone; more vibrations in a given time, higher tone also.

But neither the numbers expressing the two relative lengths of string, nor the numbers expressing the two relative rates of vibration, express any real idea of the *interval itself; how* large an interval it is, in comparison with some other, either greater or smaller; in other words a unit of measurement. Thus we do not find that the first fret of a guitar, for instance, stops one-twelfth of the string, nor one-twelfth of half the string, nor anything like either, though one-twelfth of the dual is the interval we want, the "equal," though not quite natural "semitone," of modern tuning.

I will now take up the thread which I dropped last month, and then school may be dismissed till some bright morning early in May. I wished to say something more by way of illustrating the fallacy of supposing that a musical interval is measured "by counting the vibrations," even if we have the apparatus to do it.

Suppose any tone, one, for example, whose vibration number is exactly 100; that is, 100 in a second of time, — and this would give a tone somewhere about the second G below middle C, it being the lowest G but one on almost any piano or reed-organ, new or old, or the G string of the 'cello, or an octave below the G string of the violin. Suppose now, another tone vibrating 101. Of course this is higher, and a small interval would be noticed, quite perceptible to any ear. Now suppose still another tone, 102. Would you jump to the conclusion that this is as much above the second, as the second was above the first? The jump would not, indeed, land you so very far out of the way, in this particular instance; but it would show that your means of comparing the relative magnitudes of tone intervals were simply nowhere. The vibrative *ratio* of the first little interval is, of course, 100 : 101, while that of the second is 101 : 102, a different, that is, a smaller interval — in principal, although in this case, they could not be distinguished from each other by the ear. Let us now, while we are about

it, have the real measurements of these two small, but very perceptible intervals, although they belong not to music; and let it be remembered that the dual or octave is always the unit of my decimals: —

Vibration Ratios.	Interval Magnitudes.
100 : 101	.014,356+
101 : 102	.014,214+

Here they are in black and white, and in *millionths*, the difference between them being almost exactly one-seventh of one-thousandth of a dual; and, since one-thousandth is about the ear's limit of judgment, this difference is considerably ~~within~~ beyond that limit, and imperceptible; and it is just as easy thus to find the imperceptible differences as the perceptible, and he who cannot do the one, cannot do the other, both being easy enough, however, with a simple table to start with, and a little attention.

Thus, then, while we can tell which of two ratios is the larger from the very numbers which express them, we cannot really ascertain their relative quantities without a calculation, that is, a measurement of the ratios, which is the same thing as measuring the tonia resulting from these ratios. The two intervals, just considered, if they be added together, forming one vibrating as 50 : 51, would amount to about one-third part of the "semitone," as correctly tuned on the piano or organ, which is one-twelfth of a dual.

Suppose another case, to view the matter in the light of what we all know and can hear: Take three tones having the vibration numbers 400, 500 and 600. (The 400 is about middle G, and as the perfect fourth, a trial, vibrates as 3 : 4, the C above, or "fork C," would thus be 533⅓, while that variable and arbitrary thing called in our time and country "concert pitch," is practically anywhere from this to 550, while French pitch, which has been within a year or two indorsed by the American Music Teachers' Association, is 517.3, — the fraction being accounted for from the fact that the French pitch is reckoned from A, that being 435, and the *tempered* minor third above that, or C, which is a quarter of an octave, will, as in all such cases, result in a fractional pitch number for the C, though if that minor third were true and perfect (5 : 6), the C would be 522; and this pitch for C is that of the New England Conservatory, I believe. But absolute pitch is nothing to the present subject.) These three tones, or any having the same vibration ratio, 4 : 5 : 6, form the beautiful and fundamental harmony known as the common major chord, or major triad, the ratios being 4 : 5, 5 : 6, and 4 : 6 (2 : 3). Now how do these intervals of tonia compare in magnitude? Is the tone 600 as much higher the tone 500 as this is higher than the tone 400? No; for, as most readers will doubtless see the second interval is what musicians call a minor third (5 : 6), whose magnitude is 263 034 +, while the first is what they call a major third (4 : 5), size .321,928 +, both quincals, the difference being almost exactly one-seventeenth part of a dual, although represented on the keyboard by one-twelfth, like other "semitones," greater and smaller than one-twelfth.

One more illustration, which is still more forcible: Take the dual at the pitch 100 : 200, which is about G — g (next two G's below middle octave), and suppose another tone, 300, and what do we have? Not indeed another octave higher, but an interval not very much more than half as large, namely, a trial (2 : 3), or fifth, its size being .584,962,5 +. These examples will not indeed show how to measure tonia, but I hope they are sufficient to show how *not* to imagine it can be measured.

I am merely paving my way, by feet and inches, thus far, in order to ultimately elucidate ideas of music which will seem more tangible and practical. I have to be terribly didactic, and rather against my will too, and even dogmatic and emphatic; and everybody's unfamiliarity with this ground is what makes it necessary for me to be so. I am not going into the martyr business, however. The martyrs are always in the past, not in the present, and the most of them are never heard of as such. But I am getting, and shall get, amusement and recreation from these exercises, and fondly hope that others may share them with me. And let all men, while they are about it, learn and remember that this stuff which I rattle every month, kiln-dried and neglected though it may be, is the material of true musical science, and is therefore in the direct line of progress in music in every sense of that word.

JAS. PAUL WHITE.

TONE INTERVALS.

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V.

I SUPPOSE that almost every one, even among those who are good in figures, would blunder more than once in attempting to call off the list of prime numbers between one and 100. As the well-known classification of numbers as either *prime* or *composite* is a vital element in the science of Tone Intervals, let us now glance, partly for amusement, at all the prime numbers under 100:

1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, — 26 in all.

In the next hundred are 21 primes, and in the succeeding hundreds are 16, 16, 17, 14, 16, 14, 15, and 14, making 169 prime numbers under 1,000.

In the second 1,000 are 135 primes; in the next, 127; then, 120, 119, and 114, making 784 primes under 6,000. The highest prime number known as yet, according to a note which I have made from some mathematical work, is 2,147,483,647. *

These statements will serve at least to show the curiously irregular manner in which prime numbers run, — that is, skip, hop, and jump, if that can be called running.

There is but one *even* prime number, 2, and this makes and characterizes the king of all musical intervals, whose vibration ratio is 1 : 2. — simple in its greatness, and great in its simplicity; and although called the octave in music, it is entirely independent of all

* There is no known rule as yet, for finding large primes, except by actual experiment, or verification.

scales and all other intervals, and its fitting name in science is *the dual.* All the other primes are not only odd, but *very* odd; more odd than other odd numbers.

The line of demarcation between musical and unmusical primes is between the numbers 7 and 11, the former being among the elect, and the latter, alas! a reprobate, like the infinite number of prime numbers following it. This sounds a little too Calvinistic for science, I know; but extremes sometimes do meet.

The four intervals, named the dual (1 : 2), the trial (2 : 3), the quincal (4 : 5), and the septal (4 : 7), which *make* all the music there ever has been in the world, or ever will be, are by no means equally prominent in music. The simpler the vibration ratio (among these four) the more prominent the interval. The dual is king, the trial is prime minister, the quincal is the citizen, who really makes life at all worth living, and, as for the septal, — well, she is a pot-wrestler, and yet is mighty handsome, sweet, and charming!

As for this prime number question (?), I settled it in my own mind long ago, and found, as all can find by attending to it, that 7 *is*, and that 11 *is not* a musical prime; and in fifteen years of close relationship with these matters I have never found one momentary reason or shadow of it for thinking otherwise. And yet, because writers of ability have thought differently, I have sometimes written pages and pages by way of discussing the matter, but have always ended by putting them into the fire. It seems too much like discussing an axiom. If any one wants to think that even that *prime number of ten figures* above mentioned is what makes the chord of the *diminished seventy-eleventh*, why, let him pass, — and, indeed, the man referred to in my extravagant language did pass *away* on the 28th of last October, in London — Alexander John Ellis, F. R. S., the excellent translator of Helmholtz's Sensations of Tone; and another of my correspondents also died just one week before — Henry Ward Poole; and he also was one with whom I had to differ on important points, but whose (in many respects) excellent essay on Perfect Intonation, in Silliman's Journal, 1850, helped me in 1876 to get into this investigation of intervals.

Now, there are those words of mine, — dual, trial, quincal, and septal; I shall have to harp on them a little, and then stop for this month. And, first, I like them all the better now that I have ventured into print with them. Of course many will think that it is for oddity or something else that I use such words. I say honestly that I have neither pride nor shame about them. The only thing that I do care about concerning them is, that I can use them freely and have them so well known to others that they can understand me when I use them. To a Latin scholar, however, they scarcely need explanation. The words happen to be easily and simply formed from the Latin roots, more simply than scientific terms frequently are. I only use the prerogative of all explorers of science, by making convenient and etymological terms in cases where they are necessary.

Such a rational classification of musical intervals as the four words furnish names for seems to be never dreamt of in ordinary music teaching nor interval measurement; and, indeed, there are no good and convenient books that teach, or begin to teach, music on this rational basis. JAS. PAUL WHITE.

TONE INTERVALS.

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VI.

"WHAT in sheol," said one of my critics, "do you mean by such figures as these?" pointing his finger to the terrible decimal .584,962,5 + in the January article (I.). He seemed to think, in spite of all I have attempted to say to the contrary in these articles, that the figures were intended to signify something about vibrations, instead of being merely the decimal fraction measurement (the octave interval 1 : 2 being the unit) of an interval next in prominence to the octave, called by musicians, unfortunately for science, the fifth (2 : 3). The decimal is not much different from the fraction seven-twelfths, which is .58⅓, the difference hardly amounting to anything in practical music. The fifths in our "equal temperament" tuning are theoretically just seven-twelfths.

The other two decimal measurements, however, also given in Article I., are very different from the corresponding "equal temperament" intervals, the true major-third being .321,928 + (4 : 5), and the true harmonic seventh (4 : 7) being .807,355 — ; while in our scale they are necessarily tuned so as to be one-third and five-sixths of an octave, respectively. A simple method of showing the relative amounts of these variations is given in this article. *

Now it makes no difference to nine hundred and ninety-nine persons how I get the decimals which measure the true intervals, so long as I can swear that they are true. I was not the first inventor of such lovely little calculations. Euler, the mathematician, did the same, more than a hundred years ago; although when I first made the calculations on this basis, some fifteen years ago, I did not know that Euler or any one else had preceded me in taking the octave interval as the unit of measurement. Euler's example, however, has never been followed to a large extent, writers on the subject of intervals using either no unit at all, or using a very inconvenient one. But Euler's measurement of the fifth stops at the sixth decimal figure, 2, an unlucky place, for the next figure is 5, and then there is a gap of two ciphers. So I like to take in the 5 also — out of the cold; and thus we have, in these seven decimal figures alone, a measurement of the fifth which is quite *pluperfect*, according to certain old grammars; for it is *more perfect*, by a hundred thousand times, than any human ear requires; and yet it is necessary to retain the whole seven figures for the use of science.

* See the next article (VII.)

As for the other two prime intervals, we can get along *nicely* with six figures only (millionths of an octave); but the fifth occupies such a place in music that we cannot make sufficienty nice calculations about it without seven decimal figures — which, in fact, are as good as eight or nine, on account of the two ciphers immediately following the figure 5.

Some one will ask: "If three decimal figures only, that is, .585 of an octave, are a true fifth — to the ear — why are they not true enough for science also, without *disfiguring* the paper with such an array of figures?" Let us see. For instance, there is a little interval in this science long known as the comma. Add together four perfect fifths, drop off the two octaves from the sum, and compare the remainder with a true major third. The difference is the comma (80:81). Supposing, now, some of us great scientific necessary evils think we have a good reason for wishing to know pretty accurately how many commas it would take to make an octave. We cannot possibly find out by comparing the figures signifying the respective ratios of vibration of the octave and the comma, 1:2 and 80:81. We must, of course, use the decimal measurements of two of the prime intervals, for the ratio numbers are not measurements at all. Well, then, four times .585 are 2.340. Dropping the two units, which are octaves, and comparing the remaining decimal with .322, which we are now regarding as the sufficiently true major third, we find the difference to be .018. This, then, is the size of the comma, according to this calculation, and by using this for a divisor and 1 (octave) for a dividend, we have 55.55+ as the number of commas in an octave. Now, this is not as bad as it might be; indeed, it is a much more nearly correct idea than many writers on the subject have formed, for it is only about a quarter of a comma out of the way. But this will hardly do for respectable scientific calculation; so we use the 6 and 7-figure decimals instead, and thus find the comma to be a trifle smaller than before, that is, .017.922, and that it takes 55.8 (nearly) of them to make an octave.

Again, supposing some one is downright mean enough to wish to know just about what part of an octave that little minute amount of the flattening of the fifth is, which constitutes the even temperament of modern tuning, that is, the difference between the true fifth and seven-twelfths of an octave. This latter, reduced to a decimal, is .583⅓, while the true fifth, as nearly as can be expressed by three decimal figures, is .585; and the difference is found to be just a 600th of an octave, or just a 100th part of a "whole tone" in the temporal scale. This, however, is rather erroneous, even for a conception; but by using the far more accurate 7-figure decimals, it is found that it would take about 614, that is, 613.81+ of these little "temperings" to make an octave. And this last is correct, even to the last decimal figure.

It is time now to prepare to state some things which may help to lessen the fog which hangs about these fearful decimals of mine — so to speak, for I well know that there are many, even among the more intelligent readers, who have little taste for numbers and figures, particularly in the form of decimal or any other fractions. I use these decimal measurements of the prime intervals, so far in the series, because they are the best scientific basis to work on; but, *for gaining conceptions of the subject* better units of measurement can be used, and these must be aliquot divisions of the octave interval. The late Mr. Ellis and others have made use of the twelfth part of an octave *and decimals of that* as their system of interval measurement. Mr. Ellis calls the 100th part of one of these "tempered semi-tones" a "cent," so that there are 1,200 cents in an octave. One of these cents is too small to be perceived by the ear, or, as Mr. E. expressed it in one of his letters to me, "The ear can't hear a cent." Two cents, however, can be heard, for that would be a 600th part of an octave; and this is what represents the common temperament of the fifth by this cent system of measurement, which is not very accurate, as was shown above. The only advantage of Mr. Ellis's system is that it divides the tempered semi-tone by a very round number, 100. But that does not atone for the rather serious fault of being too inaccurate for science; and this, of course, was known to him, and he sometimes had to *patch* his own system in making nice calculations. When so large a number as 1,200 is used for dividing the octave for interval measurement, it certainly ought to be a very close measurement to justify the use of so large a number.

There is a number but little more than half as large, 612, which gives both the fifth and the major third about *eight times* nearer mathematical perfection. It is by far the best ~~division~~ divisor of the octave under 1,000, or several thousands, for the three other prime intervals, and, consequently, for all others.

The best ~~division~~ divisor of the octave, in three figures, for mathematically measuring the fifth, is 665; the best for thus measuring the major third is 643; and the best for the harmonic seventh is 571; but the best for *all three* is 612. This number, then, is our octave-slicer for enabling us to gain the best idea possible of this phase of music. #

(*To be continued.*)

TONE INTERVALS.

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VII.

[I WAS slicing up the octave, as usual, in a snug little poem of two columns, more or less, for the June LEADER, and the editorial management, or some other diabolical power, caught the same spirit and cruelly sliced the chapter into two, with a "To be continued," — a great waste of ink in these scarce times. Now I wish the readers of this paper to under-

$\frac{389}{665}$ of an oct. falls short of the true fifth only about $\frac{1}{17200}$ of a nil! and $\frac{267}{643}$ and $\frac{461}{571}$ oct. are respectively within about $\frac{1}{4500}$ and $\frac{1}{2700}$ nil of the true major third and harmonic seventh. In the 612-division (nil division) they are $\frac{1}{338}$ (ex), $\frac{1}{50}$ (def.), and $\frac{1}{10}$ (def.) of a nil respectively. F. P. W.

stand that, at the place where the editorial gash was made, I was dogmatizing faithfully about the 612-division of the octave, the best *mental* division of the octave possible for gaining a good conception of tone intervals as they relate to our common, "equally tempered" system of practical music. And I have here numbered the remainder of the article VII. in order to make the numbers agree with the month numbers of the year 1891. I went on to say of the 612-system, and other things, as follows: —]

The natural musical intervals which are *personally* concerned in this or any other aliquot division of the grand unit, are various; but let no one believe for a moment that it is necessary — any more than it is convenient — to consult *all* musical intervals to find out how any one division of the octave is going to affect them all. If we would know how just our aliquot slicing of the octave is going to be to *all* musical intervals, we have three individuals to consult, and three only. We have not to consult so prominent an interval as the fourth, for that belongs to the *trial* family, and vibrates as 3:4; and it is nothing more nor less than the difference between the *trial* (2:3) and the *dual* (1:2). Nor have we to consult their honors, the sixths, nor the minor third, nor seconds, nor sevenths, save one, nor chromatics, nor commas, nor dieses, nor the skhisma, nor the diaskhisma, nor the ~~apotome,~~ nor limmas, nor various other difference intervals which some good thinkers in different ages have named and — left to rot! And let them rot, the most of them. We shall see all these little intervals, however; but we care little about such old and more or less useless names.

apotome

No; if we slice up the dual interval, either into coarse or fine equal slices, and wish to know how such a course of proceeding, either in imagination or in fact, will affect *music*, either as a science or an art, the only intervals to be consulted are the fifth, the major third, and the harmonic seventh, about which I have said so much, and very little, as yet, about any other intervals; for these four being in tune, everything else is in tune, and *vice versa*. *For* there are no other vibrational *prime numbers* in any true musical interval, which are not in these four, — and the primes are 2, 3, 5, and 7; this last forming the septal, or harmonic seventh (4:7), which is a much lower tuning than the "flat seventh" on instruments, but not the "diminished seventh," which, on the instrument, is a sixth.

and rather indefinite

The same as

The various "steps" of our scales are merely the differences between a few harmonious intervals, the last step of the "diatonic scale," for example, being exactly the difference between a true major third (the quincal, ~~4:7~~) and the true fourth (a trial 3:4), that difference being, vibrationally, 15:16.

4:5

It may interest some to state here that to find what ratio is the *difference* between two ratios we have only to perform the same operation as in dividing one fraction by another; and to find what ratio is the *sum* of two others, we merely proceed, as in multiplying one fraction by another.

although a ratio is not a fraction.

As to the actual magnitude of the above mentioned "semitone," it can easily be found by substracting the decimal measurement of the fifth from ~~(fig. 1)~~, (the octave), to get the fourth, and then subtracting the true major third from that, and the result is .093,109+; and it is found by division that it would take almost exactly $10^3/_4$ of these to amount to an octave, which shows that this little step is rather larger than one-twelfth octave, as all kinds of semitones are tuned.

from unity (the octave)

This will suffice for an example. Now we return to our *surgical* operations. We have first cut the octave — mentally — into ten-millionths; these are the decimal measurements, and I might easily give them in millionths of millionths if necessary. This ten-million system is the sufficiently accurate and convenient scientific standard with which to compare all coarser methods of octave-splitting — and, by the by, if a student cannot split even a hair, and very approximately in the middle, in measuring tone intervals, and see the advantage of being able to do it, then he cannot split or understand much of anything about it.

There are two aliquot divisions of the octave which will occupy the rest of this paper, that of 12 and that of 612; and if I do not say enough about them here I may supply some deficiencies hereafter.

Now, there are various aliquot divisions of the octave which are more or less interesting, but very few of them are of enough use, either in science or in art, to deserve *names*. But 612-division, and the 12-division, are exceptions, both of them being invaluable in science, and the latter invaluable in practical music; and the artificial units which both make should not be forever nameless.

the

I called the first of these a *nil* in the November number of this paper, and shall probably abide by the name. A 612th of an octave is so near to nothing, to the ear, that I call it that and "done with it," the word *nil* meaning *nothing;* and it is the unit of the *nil system*. The number is a multiple of 12, as well as Mr. Ellis's 1200 is. The "equal semitone" is therefore 51 nils, and of course all the other "tempered" intervals are from two to eleven times this, in nils.

Our prime intervals are measured in the nil system by 612, 358, 197, and 494 nils, while seven times 51 (357, the tempered fifth) is one nil short; four times 51 (204, the tempered major third) is 7 nils too wide; and ten times 51 (510, the tempered harmonic seventh,) is 16 nils too wide.

Once more I repeat that this nil system furnishes the best conception possible of the arithmetical ~~phrase~~ of our common scale as correctly "tempered," and I am sorry to say that I seem to be the first to publish it, to declare its value to science, and to give it a name. There is no other system possible which can compare with it in usefulness for this purpose.

— phrase

From this simple showing, many clouds and much fog can be lifted from this badly understood subject of intervals, and the "temperament" of the scale. And in my future articles I shall use the nil system freely instead of the decimals of an octave, which,

however, are the *source* of all nice scientific calculations. But see, and behold how much more economical are the nil measures for general teaching. For instance, the comma (80:-81) in the decimals is .017,922; in nils it is simply 11. The Pythagorean comma, (difference between 12 perfect fifths and the starting point, octaves being disregarded) is 12 nils, while in decimals of an octave it is .019,55. The little interval called a skhisma (the difference between eight perfect fifths and a minor sixth, octaves being disregarded,) is one nil, the temperament of the fifth being almost mathematically the same thing, so that both are expressed by one nil. Now the skhisma, in octave decimals, is .001,-628, 1 +, and the aforesaid flattening of the fifth is .001,629,2—. The nil, which is .001,-634—, thus does nicely for both. The septal comma (63 : 64) is 14 nils, and the diesis (difference between three true major thirds and an octave) is 21 nils — one-third of which, or 7 nils, is placed upon the shoulders of every major-third in the common tempered scale. These simple numbers are certainly much more convenient and intelligible than our excellent and extremely accurate decimals of the octave.

The nil measurements of the "untempered" scales may come up in the next article, though some of my ingenious readers will perhaps before then have found most of them by simply subtracting the nil measurements of the more harmonious intervals. I shall have more to say too, hereafter, about the mathematical nicety of the nil system, for, indeed, I am churning out the present article without access to my well-filled notebooks, which are one hundred miles away.

Now our common 12-system of practical music will feel ashamed without a name, also; and it is certainly high time it had one.

Long before the English language existed, the twelfth part of anything was called *uncia*. Our word *ounce*, by a roundabout way, through the French, is derived from it; and there may have been a now obsolete English word, *unce*, derived directly from the Latin one. It is perfectly etymological, however; and now I suppose the only thing I shall suffer (?) if I call the twelfth part of an octave an *unce*, is being denominated an everlasting pedant. Nevertheless, let all beware, for I am armed with the word *unce*, and it only wants a d prefixed to do good service in defending myself against my enemies!

Why, is it not strange that we have not previously heard of the *unce* or *uncial* system, when we have been using it right along, more or less, for over a hundred years!

TABLE I.

51 nils make one Unce.
12 unces " " Dual.

TABLE II.

612 nils make one Dual (1 : 2).
358 " " " Trial (2 : 3).
197 " " " Quincal (4 : 5).
494 " " " Septal (4 : 7).

TABLE III.

612 nils make one Octave.
357 " " " (temp.) Fifth.
204 " " " (temp.) Major Third.
510 " " " (temp.) H'monic Seventh.

We are now getting into a condition to begin to state how our common uncial system is to be regarded. We are unfolding some truths which will prevent us, on the one hand, from imagining, as some theorists have done, that our uncial scale is destined to destruction because of its inharmoniousness (it never can die — it is invaluable); and, on the other hand, from believing it to be all music itself, for it certainly is an evil and mischievous thing in more respects than one. It is hoped that in the course of a century or two (no joke intended) — it will take a century, at least — it will be possible, by ordinary means of study, to learn something about musical intonation.

There is *so much* at present to keep the mist about us! — ignorance, indifference, books, and other publications — which undo and tear down sometimes faster than the present force can do and build up.

JAS. PAUL WHITE.

TONE INTERVALS.

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VIII.

I SUPPOSE that among the numerous faults which are found with me in these exercises is the fact that they do not contain any acoustical science; but I am not writing on acoustics. Some of my readers read acoustics, of course; and they doubtless think me erratic because I do not preach the gospel of acoustics to them, especially as acoustical writers have rather abused this arithmetical phase of musical science which I delight to honor. In their ignorance of it they have knocked it into pi; they have despised it as a thing not worth caring for; they have put their readers, who would have searched for it, off the track, or so hedged up their way that they have become blissfully contented with a little of that sort of musical science which dwells in upper partial tones and beats — all of which is very good and useful in its place, but gives us no conception of the structure of music; so that when a fellow would fain be understood a little on the subject of Tone Intervals, he has to begin by showing the evident fact that this is a separate science of itself, and that acoustics, especially as that now is, does not at all teach Tone Intervals, or toniametry, as I have called it.

Hence, my quadruple classification of intervals, the four names, and the numerical measurements, which are the most systematic and the simplest possible for real science, seem somewhat strange and unheard-of to perhaps the large majority of those who are so good as to read my articles.

Then, again, some are already in affliction because I have not yet said anything about such trite matters as the non-identity of G sharp and A flat. They believe that that is something just in my line, and that I ought to attend to it at once! This, being a splitting of hairs (I being the hair-splitter), should be discussed in due "musical" form, with a specious handling of our beautiful chromatic scale nomenclature! And when the thing should be done, every one would of course be very much edified and very wise in the intricacies of this wonderful and inexplicable thing, "the musical scale!" Now this G-sharp and A-flat business is not a hair-splitting at all, but a "chestnut," and just about as indefinite; and if my dearest readers will only be good and quiet till they acquire a few more grains of solid toniametry, they can then begin to laugh, all on their own hook, at any such phantoms as W sharp and Y flat!

Am I making sport of things which are greatly respected? Do I laugh at, and make others laugh at, "music" and the musical nomenclature which all the musical world — and their aunts and cousins — worship? And am I also indiscreet enough to give even the science of acoustics, in its present attitude, a nudge under the chin?

Just so.

Some one had to do it, and in the process of time the duty and pleasure was mine. The science of Tone Intervals has been under wholesale abuse and in complete disgrace long enough.

And yet I am not so foolish as to be at war with any individuals. I can at least partly understand why these things have remained so during this century, and I have a strong conjecture that in the twentieth century there will be a marked change in this matter; and, poor stick that I am, I expect to leave something better than the present series of papers as a material aid in forming, disseminating, and utilizing the true and excellent science — a science which will not only give the pleasure of understanding music in reality, but also give the pleasure of making it more musical and of hearing it with more satisfaction.

As to the few new words introduced in this series, I would say here that I have no idea of instituting a new nomenclature for practical music, nor of trying to mend the present one. It is not my business at all. Our names of intervals, derived from "the scale," are indeed very vague and confusing for such use as mine — and yours, if you wish to understand things; but I shall have to use "thirds," "fifths," etc., to some extent, and may even sometimes *condescend* to such unscientific lingo as "sharps" and "flats;" but my generic names of the four intervals which alone make all the music there is or ever can be in the world will maintain their rule, as that which they signify has always done and always will do.

After all, but few new names have been introduced, and these will probably about suffice for the present series, at least; and if we are not afraid of their novelty, they will be more and more invaluable to us.

Glossary.

Dual — the interval whose vibrations are as 1 : 2 (called the octave). It is itself the grand unit of all interval measurement, but in the nil system it is 612 nils, and in the uncial system it is 12 unces.

Trial — the interval whose vibrations are as 2 : 3 (called the fifth). Magnitude, .584,962. 5 + of a dual; but in the nil system of measurement it is 358 nils. Our common uncial system lessens it one nil by giving it as 7 unces, or 357 nils.

Quincal — the interval whose vibrations are as 4 : 5 (called the major third). It measures .321,928 + of a dual, or 197 nils. It is represented by 4 unces; that is, 204 nils, which make it 7 nils too large.

Septal — the interval whose vibrations are in the proportion of 4 : 7. Called by musicians — well, they don't call it anything, though they use it constantly with more or less impurity. They have in mind only one "flat seventh," which does duty for three different intervals, the septal being the smallest interval of the three, the other two vibrating as 9 : 16, and 5 : 9, the former being a trial and the latter a quincal; but the septal is the most harmonious to the ear of all "sevenths," as its very ratio numbers, 4 and 7, would suggest, especially as one of them is a dual number; for, as a general rule, the smaller the vibrational ratio numbers the more harmonious the interval. Helmholtz calls this interval the "sub-minor seventh." It has also been called the "prime seventh," the "harmonic seventh," etc.; and, so long as we use the scale name of "seventh" at all, — which confuses the idea of the prime number 7, characterizing its vibrational ratio, with the *seventh* place in the scale of the dominant, — it is little matter what we do call it. It is the septal anyhow, and, though in a less prominent way than the three other prime intervals, but not less really, forms a family or race of musical intervals, all of which have 7 or its multiples in their vibrational ratio.) Its magnitude is .807,355, or 494 nils; but it is represented — and very badly too — in the uncial system by 10 unces, which are 510 nils; and this makes it 16 nils too large. The other two "flat sevenths" afore-mentioned are also represented by 10 unces, and one of them (9 : 16), being 508 nils, is only 2 nils out, and the other (5 : 9), being 519 nils, is 9 nils out, the common system of "equal temperament" thus giving these two latter sevenths much better than it does the septal.

Undecal, *tredecal*, *septendecal*, *novendecal*, 23*al*, 29*al*, etc. (analogous to the terms *octavo*, *duodecimo*, 18*mo*, 24*mo*, etc.), would be the names of intervals characterized by higher prime numbers than 7. But as these are entirely *unmusical* intervals, there is little use for such words. All these useless intervals, however, are as easily measured as the others by the dual decimals, the undecal (8 : 11), for instance, being .459 + of a dual; and it would be something like half a "semitone" larger than a fourth. *

Tonia — the difference between two tones, a magnitude which can be measured, as approximately as we desire, by any unit we

* Of course any prime interval, or within belonging to music or not has to be calculated (or reckoned) in definitely of any other prime interval or other. This is found for the octave decimals by dividing log. 11 by log. 2, and cutting off the resulting units.

may choose. Thus we say, an interval of tonia, as we would an interval of time or of space.

Toniametry — the branch of mathematics which treats of measuring tonia or tone intervals, including their classification. This science is to be in the future — what it doth not yet appear. ※

Unce — a twelfth part of a dual; the unit of the modern practical system, especially of the present century (and of all centuries following, for a large part of music). It forms a system which does considerable violence to quincals and septals.

Nil — a six hundred and twelfth part of a dual, the unit of the nil system of interval measurement. This is the best aliquot division of the octave possible for gaining a good and sufficiently true conception of true tone intervals as they relate to our common practical uncial system.

This will do for the glossary, the last two defined words of which give names to two artificial divisions of the octave. It will be asked if the unce, our "equal temperament semi-tone," is indeed an artificial and unnatural interval. Certainly it is, and so are two or more of them added together, till we come to twelve, which fill a true octave.

If we could play true and harmonious music in the various progressions on our instruments,— which means, as a general rule, merely playing the four prime intervals in tune,— there would be in the combined melodies usually five sorts of "semitones," three sorts of "whole tones," and four kinds of harmonious thirds, as well as several inharmonious but still melodic ones, saying nothing of larger intervals.

The smallest natural and melodic "semitone" (24 : 25, the difference between the true major and the true minor thirds, 4 : 5 and 5 : 6) is only 36 nils. It is a quincal. The next in size is 43 nils (20 : 21, the difference between the major sixth, 3 : 5, and the septal, 4 : 7), and it is a septal. The next in size is 47 nils (128 : 135 — approximately 18 : 19, though the prime 19, of course, does not really belong to musical ratios), and this is the difference between a certain "whole tone" (8 : 9, a trial) and a certain "semitone" (15 : 16). It is a quincal. Then there is this same (15 : 16), the difference between a true major third and a true fourth (3 : 4); and it is the so-called "diatonic semitone" of 57 nils, the last step of either major or minor scale. Then there is the still larger (14 : 15), 61 nils, the difference between the septal (4 : 7) and a certain major seventh which is the inversion of this aforesaid 15 : 16, namely, 8 : 15. The unce, which represents them all, is 51 nils, one-twelfth of 612.

The major seconds (7 : 8, 8 : 9, and 9 : 10) are 118, 104, and 93 nils. The first is simply the inversion of the septal. The next is a fifth doubled minus the octave, or, which is the same thing, the interval left after ascending a fifth and descending a fourth. It is a trial. The other is the difference between this last and a major third, and it is a quincal. All these are represented by two unces, which are 102 nils.

The four harmonious thirds (represented by only two roughly harmonious ones) are 222, 197, 161, and 136 nils (7 : 9, 4 : 5, 5 : 6, and 6 : 7).

I do not imagine that musicians will generally believe the truth of these truly scientific statements, especially concerning the four harmonious thirds; and I am perfectly familiar with the real reasons why, from their standpoint, they are thus sceptical; and one is, that two of these thirds, the largest and the smallest, are septals, a family of intervals which can bear — and indeed they have to bear — more abuse than can the real major and minor thirds, which are quincals. Our uncial system lengthens the major third 7 nils and shortens the minor third 8 nils; and although that does not meet the gap to the two septal thirds half-way — that gap being 25 nils — it thus makes the tempered major third of four unces answer for both the large thirds, and the tempered minor third of three unces answer for both the small ones.

The four harmonious thirds appear to the best advantage in the complete five-toned harmony (4 : 5 : 6 : 7 : 9), the sound of which always surprises a musical ear when it first hears it and knows what it hears, so to speak. It appears on the instrument as a major triad succeeded by a minor triad; but the last three tones are not a minor triad at all, for if we should make it one — in natural tune — that is, by making the first third really minor (5 : 6) and the second really major (4 : 5), and leave the major triad (4 : 5 : 6) also in natural tune, the whole combination would sound much worse than in its common "tempered" state, on account of the fourth tone (7), which would be so very far from its normal pitch — actually 25 nils, or about half an unce, too high. This illustrates the fact that, however they appear on the instrument, two or more succeeding minor thirds never really occur in a harmony. The little septal third (6 : 7) always has a part in the matter whenever a minor third, on the instrument, seems to immediately succeed another in a chord.

The above five-toned harmony, called the major ninth, is a septal combination; and so also is the diminished seventh, which *appears* to be three minor thirds on the instrument. There is in it, however, one little third (6 : 7), which makes the whole combination septal.

It is objected to septal intervals — by those, of course, who do not know how much septals have to do with all music — that they sometimes make bad melody, however beautifully they sound in harmony. There is some truth in this objection, on account of a certain fact which will come up later; but the case will be found to be quite different from what is supposed.

It is probable that my next paper may be a trifle "interesting," for it will show where scales come from. A shower of "sharps" and "flats" will *naturally* follow, either in that or the next article.

Then, after that storm abates (after equinox, probably), I may touch upon the aliquot division of 53, which one or two of us in the past few years have utilized to some extent in reed instruments, which render slow, solid chord music at least in beautiful harmony; but it is not at all the purpose of these articles either to advertise or very particularly describe my instrument, the harmon, which is not a business enterprise. The 53-system, which is easily enough tuned, in the ordinary organ reeds (I have never yet tuned this cycle in organ pipes), by understanding the law of beats, reduces the above five semitones to three, they being 3, 4, and 5 53ds each; and it also recognizes the four harmonious thirds, greatly to the ear's satisfaction.

※ All this of course sounds rather pedantic, like much else in these articles. But I care nothing for the fate of these words of mine, the ideas, however, ought to live.

TONE INTERVALS.

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IX.

In treating of musical scales, which I shall do in this and also in another article, it must be understood that I am not a believer in the possibility of always executing, in their mathematical purity, the true intervals of these scales which I shall attempt here to describe. But I do not think it is necessary that music should be rendered, either by voices or by artificial instruments, so discordantly and unmelodiously as it usually is, and I am sure it would be much improved in this respect by the kind of musical culture which would accompany an understanding of some leading truths in the arithmetical phase of music. I do not say, however, that musical people at present have the books and teachers by which they could gain — by anything more than a moral possibility — such education.

All that I can hope to do in this and a following chapter is to present, with but little qualification, the substantial facts which will enable those who think of these things to form right conceptions, and to understand how it is and why it is that the octave interval divided into no more and no less than twelve equal parts, happens to form a musical system, and one in which the harmony and melody is even approximately true.

The only possible materials of all musical scales and of all melody, are the four vibrational elements dwelt upon in the former chapters, namely, the dual, the trial, the quincal, and the septal; and the various *steps* of scales, each belong to one of the four families of musical intervals, which take the name of their progenitors. The step, for instance, formed by the vibrational ratio 8 : 9 is a trial interval, for the ratio contains no higher prime number than 3; while the step 9 : 10, although nearly the same in length, is a quincal, for its highest prime is 5. The short step, 15 : 16, is also a quincal, but the still shorter one, 20 : 21, is a septal, while a still shorter, 24 : 25, is a quincal again, etc.

But we are too fast; let us begin in better order.

I wish I could define, in a very few words, just what I mean by a harmony; but the most excellent reader must supply by his own wits what my words fail to express. Harmonies have three, and perhaps never more than five, tones, exclusive of octaves. They are also called chords or concords. Two harmonies vibrating, for instance, as 4 : 5 : 6 and 10 : 12 : 15 : 18, are not equally harmonious, even when perfectly tuned. All harmonies may be perfect in their kind, but no two are equal in their harmony. We may say a smooth or superior harmony, and a rough or inferior harmony. The words "consonant" and "dissonant" are used; but if I could have my way I would reserve both the words *dissonant* and *discordant* solely for chords perceptibly out of tune, and combinations which have no approach to musical relations of any kind, and which are to be got rid of as much as possible in music. Inferior harmonies (that is, those which have the larger numbers in their vibrational ratios, though no *prime* numbers higher than 7) are of indispensable use in certain relations with the more harmonious harmonies; but discords, as I would thus define the word, are the enemies of music, and therefore do not make it sound better. We never choose to pinch our fingers in the crack of the door, in order that they may feel better when the pain is gone.

There are a few leading and more or less smooth harmonies which make the musical scales. They each have a radical or root tone, which is vibrationally either 1 or some dual number, 2, 4, 8, etc.; and these harmonies are the major triad (4 : 5 : 6), the minor triad (10 : 12 : 15), and the septal tetrad (4 : 5 : 6 : 7). There is still another, as we shall see farther on, a pentad (4 : 5 : 6 : 7 : 9), which deserves a place among the others, as helping to build scales, although it contains no prime element which is not in the preceding one. In all these harmonies, the first or lowest tone is the root, the ratio number being dual; and it is to be noticed that the first and third tone of each one form a true fifth, the ratios being simply 2 : 3; and the reason for using larger numbers is to show the combined ratios of each whole harmony in whole numbers, thus avoiding fractions.

It should be further noticed that the minor triad is formed of precisely the same intervals as those of the major triad, a minor third (5 : 6), a major third (4 : 5), and a fifth (2 : 3); but the natural, direct or harmonic order is inverted, so that the resulting combined ratio is in larger numbers. For this reason (and also for acoustical reasons, which are out of the non-acoustical line of these articles) the minor triad is not quite as harmonious as the major, and the ear corroborates the scientific facts when both triads are in perfect tune, not "tempered," as on the correctly tuned piano or organ. The two triads are both quincal, five being their highest vibrational prime; and it is, in fact, the quincal element in music which mainly gives character and beauty to its structure.

If we add dual tones nothing essential accrues to the character of these or any other harmonies; and it will be readily understood by most of my interested readers that harmonies may be "inverted" in various ways, without losing their general characters, by multiplying or dividing certain numbers in their ratios by 2, thus raising or lowering their tones an octave, or dual.

Notice here that if we add the octave tone to the septal tetrad, making it 4 : 5 : 6 : 7 : 8, this one harmony alone will contain all the vibrational elements of music, for it has the dual (4 : 8, or 1 : 2), the trial (4 : 6, or 2 : 3), the quincal (4 : 5), and the septal (4 : 7).

Let us make some *ladders* out of these harmonies, the word *scale* being from the

Latin *scala*, a ladder; and the word *step*, as an interval of music, is therefore quite as proper as the word *scale*; and even *half-step* is less objectionable than *tone* and *semi-tone* for the same intervals. But custom sanctions curious things, and at the same time leaves many good and sensible things in the shade.

I will begin with a very simple ladder, and will use the time-honored solfeggio names for the "rounds." The major triad itself, with the octave tone added, will make a very good ladder or scale to start with: do, mi, sol, do (4 : 5 : 6 : 8). The distances between the rounds are 197, 161, and 254 nils. The steps are rather long, it is true, but very firm; and notice the fact that the first step — a major third, which is of course a very harmonious interval — is, after all, the least harmonious if any which are formed with the lowest tone. The next step completes a fifth from the ground tone (2 : 3); and then the vocalist bounds up directly to the top round, which makes a still more harmonious interval with the lowest round, it being the octave (1 : 2). And here he pauses a moment, well satisfied with his three jumps; for each succeeding bound brought him more and more into harmony with the tone he started with.

But it will be said that this is not the diatonic scale. Now, as the word *diatonic* signifies *through the tones*, it will be rather impracticable to make a diatonic scale, strictly speaking. But let us see if we can make one *more diatonic* than the above. We can do it by combining with it another simple ladder just like it. But in order to have the combined result still a musical scale of tones, we must place the first round of this second major triad ladder in the right place. We might place it so that the first round would coincide with the last round of the first ladder, but this would only double the length of the ladder, without making any essential additions to its tonal character.

We therefore look for the most harmonious division point in the first ladder, and that we now all know is at the tone sol, for it vibrates with the lower do, in the simple ratio of 2 : 3, the simplest ratio next to 1 : 2 itself. Against sol therefore we place the first round of the second ladder, which is just like the first, but we call it sol, si, re, — and no matter about the top round, the high sol; and even the re is higher than the top of the first ladder, so we will invert the triad by dropping re an octave, making the second triad ladder re, sol, si (3 : 4 : 5).

The two ladders thus combined are now do, re, mi, sol, si, do; they vibrate as 8 : 9 : 10 : 12 : 15 : 16; and the steps measure 104, 93, 161, 197, and 57 nils, and among these steps are still a minor third and a major third. From this very incomplete and unsatisfactory scale, however, we can begin to get an idea of the nature of melody, it being the offspring of the most harmonious harmonies; or, to drop the figure, the little intervals of melody and scales are but the differences between the harmonious — that is, the more harmonious — intervals.

We have now got the two most important component parts of our ladder, and got them rightly put together. Those two rather wide jumps, however, mi – ~~si~~, and sol – si, suggest that two rounds may be wanting. But let no man profane it at random. We must "interrogate nature" to find the exact places for the two tone rounds. We must do somewhat as we did in adding the second ladder, that is, look for the most harmonious point remaining which cuts the octave interval of the first ladder. We might suppose that the only remaining round of the first ladder, mi, is its next most harmonious division point; but it is not so. mi – Sol

We can divide the octave interval naturally not only into a fifth and a fourth (2 : 3 : 4), but also, of course, into a fourth and a fifth (3 : 4 : 6). This tone represented here vibrationally by 4 is a point which is the second place of honor in the natural division of the dual into two parts; and this tone, which is fa, will be the first round of the third ladder precisely like the others, and we call it fa, la, do, fa. But the highest tone we do not need; and now we have in the three combined ladders, do, re, mi, fa, sol, la, si, do; and the steps are vibrationally 8 : 9 : 10 : $10\frac{2}{3}$: 12 : $12\frac{1}{3}$: 15 : 16, or, avoiding fractions, 24 : 27 : 30 : 32 : 36 : 40 : 45 : 48. And yet there are really but three different ratios among these steps, and therefore but three different lengths of steps; for 10 : $10\frac{2}{3}$ is 15 : 16, like the last step, and $10\frac{2}{3}$: 12 and $13\frac{1}{3}$: 15 are each 8 : 9, like the first, and 12 : $13\frac{1}{3}$ is 9 : 10, like the second step; and the respective measurements, or intervals of tonia, are 104, 93, 57, 104, 93, 104, and 57 nils. The "equal temperament," or duodecimal scale, gives them respectivly 102, 102, 51, 102, 102, 102, and 51 nils, thus making two of the steps really just half as long as the five others, and every major third 204, instead of the true 197 nils, and every fifth 357, instead of the true 358 nils. And this tempered scale also represents several other natural forms of the so-called diatonic major scale, one form of which (and on the whole the most melodic and useful of any one form of it) I have attempted to explain above. But some suppose that this scale, thus composed of three major triads in perfect tune, contains all the tones required for rendering any piece of music in full and true harmony which has no "accidentals" in it. And even some who have speciously made considerable progress in this branch and have written upon it, do not seem to be aware of how far the said error deviates from the truth. About the only tone in the whole seven which is not liable to vary from the above "true" form — either 11 or 14 nils! — is the seventh tone, si; 11 nils being the well known quincal comma (80 : 81), and 14 nils a comma not as well known (63 : 64), a septal.

The above leading form does not contain the septal element at all, a trial interval of the second degree (9 : 16) being used for it. This is 14 nils larger than the true septal (4 : 7), and the tempered scale makes it even two nils larger still. Neither does this form con-

tain, for its first step, the lesser major step (9: 10), which is frequently wanted to render a certain minor triad in tune. This is re, fa, la, and, as it stands in the above "true" scale, it is not in tune by 11 nils.

But I can thus only touch upon these things. I have become so familiar with the whole ground myself, that I do not find great satisfaction in the little that I can — especially in a periodical — set forth.

Yet I think it is better even to do this, in a consciously imperfect manner, than to make people believe that they have, by a little mere reading, "got the whole business," when they have not got even a true shadow of it. It may not exactly please most people to show them some of their ignorance (and we are all ignorant of most things), but it is more honest thus to do, at any rate, than to call their ignorance wisdom and knowledge, though it may be bliss.

And yet the question may fairly be raised whether such honesty is honest, since it robs certain persons of what — to them — is bliss.

TONE INTERVALS.

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X.

In the last number we caught a glimpse of how a "diatonic" scale is evolved from the triad 4 : 5 : 6, called the common major chord; and this, including of course its inversions and extensions, is the most harmonious harmony of more than two tones in all music. Its two inversions are vibrationally as 3 : 4 : 5 and 5 : 6 : 8, and some of its extended forms vibrate as 1 : 3 : 5, 1 : 3 : 10, 1 : 5 : 6, 1 : 6 : 10, 5 : 12 : 32, etc. These its ratio numbers must consist of, first, either 1, 2, or some power of 2, for a root of the chord; then there must be either the two next higher prime numbers, 3 and 5, or else these numbers multiplied by 2 or some power of 2. In any of these cases the three numbers will form the vibrational ratio of this great leading harmony of music. In the limits of one octave it has but three forms. It is more or less pleasing to the ear in all forms within the limits of perceptible tonia, especially with rich qualities of tone; but in the extended and reënforced form, 2 : 3 : 5 : 8, it is a little the best of all. Here is first a fifth, 2 : 3, then a major sixth, 3 : 5, followed by a minor sixth, 5 : 8, the whole interval, 2 : 8 or 1 : 4, being two octaves.

But if we strike this chord on the evenly tempered piano or organ, in any of its forms and in any key, we do not get the real loveliness of the harmony, owing almost entirely to the quincal tone (vibrationally 5, or that multiplied once or more by 2), which in that even scale is 7 nils too high to vibrate thus in its own true ratio, the nil system of measuring tonia making the octave 612 nils. Tuners have thus great advantages over others in being able easily to test and become familiar with such facts, *especially if they know, to begin with, what the facts are.*

Three of these triads combined in the most harmonious manner possible, as we saw in the last article, that is, by placing the root of the second and third triads at a trial (2 : 3) above and below, respectively, the root of the first or central triad — the chords being so inverted as to fall in one octave — result in one style of that "popular melody" called by musicians the diatonic major scale. I say "one style," and it is probably worthy of being called style number one, for it is rather more melodic, on the whole, than other styles of the untempered diatonic major scale to be described.

Now I will spare the reader from many particulars concerning these scale styles, both major styles and minor styles; and let us be thankful, by the way, that the scientific (though popularly written) character of these articles dispenses us from the obligation of gravely dwelling upon the "eight tonal modes" of antiquity or mediæval times (for there is neither science nor music in any such idea); nor upon the scale nonsense which the late good Mr. Ellis has taken so great pains to gather from eastern and other far-off nations with which to disfigure the pages of his last edition (London, 1885) of Helmholtz. Still, a man who could admit the number 17 as a *musical* vibrational prime, skipping over the two next lower primes, 13 and 11, and not giving the really musical prime 7 as much honor as it deserves, in the instrumental and vocal music of all peoples, would be very likely to collect some rubbish wholly unfit to publish to the world in so neat and costly a form in connection with what is truly valuable in musical lore. There is no "17" at all in the vibrational ratio of what is called the chord of the diminished seventh, this being one of the septal tetrads, and a very curious one, but not to be described here.

The musical scales, however, are made from the four best harmonies, namely, those vibrating as 4 : 5 : 6, 4 : 5 : 6 : 7, 4 : 5 : 6 : 7 : 9, and 10 : 12 : 15, this last (called the minor triad) being nothing but the major triad *reversed.* The major triad (4 : 5 : 6) is so wonderfully simple and harmonious that we can, of it alone, make a remarkable musical scale, as has been shown. But we cannot make much of a musical scale with either of the others alone. Yet we shall see what can be very easily done in that line with the major triad combined with the septal pentad (4 : 5 : 6 : 7 : 9), and without a repetition of either. Call the first harmony do, mi, sol, and the other sol, si, re, fa, la; the combined result, reduced to one octave, is do, re, mi, fa, sol, la, si, do. But the fa and the la are not the same by considerable as they were in the first style. The fa is 14 nils lower thus making the fourth from do false by that amount; and the la is 11 nils higher, thus making the sixth from do false by that amount. This scale, although giving no subdominant chord (fit to be heard), gives a

most beautiful dominant chord of the ninth, that being one of its two component parts; but the first style could not give this at all in tune. To change this last style of scale into the former, we have only to change the very large third (7 : 9), called by Helmholtz "supermajor," into a major third (4 : 5), the two intervals measuring 222 and 197 nils respectively; and we cut off 14 nils from its lower side and 11 nils from its upper, the whole difference between the two-thirds being 25 nils, or about half an unci (51 nils). But this difference of about half a "semitone" never occurs in music *in one lump*.

The tempered major third, of 204 nils, or 4 unces, answers for both these thirds in the uncial scale, just as the tempered minor third of 153 nils, or 3 unces, answers for both the true minor third of 161 nils and the little ("subminor") third of 136 nils (5 : 6 and 6 : 7).

These four kinds of thirds and this kind of a scale, made of only two harmonies, are what we have all heard times without number in many accordions, and even in their miniature mouth instrument, — the air when passing one way producing the major triad, and when passing the other way, this five-toned septal chord, though the root, sol, comes from the bass accompaniment at the other end of the bellows, and the harmonica cannot give this root *with* the four other tones. As for the subdominant chord, these popular little instruments cannot give it; and the reason why the two-toned imitation of it (the third 7 : 9) which they do give sounds at all passable, is, that the first of the scale cannot be given with it, from the nature of the instrument. If it did, such a subdominant chord would be simply horrible. The fourth of this accordion scale (as it is most often ~~turned,~~ I think) forms the true septal (4 : 7) from the dominant, but would make a terribly flat fourth on the tonic, which fortunately it never can do, for the performer cannot draw and press nor suck and blow, *very conveniently*, at the same moment.

But these two are not all the styles of an untempered major scale.

I must here reveal one of my "finds" of the year 1876. I had not read many works on this subject then, having been both helped and led astray by the little I had read. What I had gained by my own thinking — I will say it — was true. Well, I had found out, one way and another not only about the musical vibrational prime numbers, 1, 2, 3, 5, and 7, which make all music, but also that the three best harmonies are the major and the minor triads, and the four-toned harmony, or septal tetrad, 4 : 5 : 6 : 7. I had never happened at that time to see, or at least to exactly take in, any explanation whatever of what makes the "diatonic" scale. But I was still to some extent possessed of the common mischievous idea that such a scale is something fundamental in music, and therefore I must needs find its rationale; and I found it by combining the three harmonies aforesaid. Taking first the major triad (4 : 5 : 6), which is a little the most harmonious of the three, I next hitched in the chord 4 : 5 : 6 : 7, putting its root (4) at the fifth of the major triad (6), and this makes six tones; and, lastly, to make a long story short, found that the minor triad is the most closely related to the major when the major third in both coincide. And indeed the minor triad in this connection with the tonic major chord is happily called by musicians the *relative* minor chord. This, then, gives a sixth to the scale, and makes out the seven tones; and when these two last harmonies are appropriately inverted, a "diatonic" scale appears all in one octave. I was quite happy in this acquisition, and probably made a fool of myself several times about it to friends, acquaintances, and those I was not acquainted with. And even when I found soon after, in reading, that "the scale" had been explained by making it of three major triads as presented in the former article, and that Mr. H. W. Poole had written about another style of it in *Silliman's Journal*, that is, the second style above described, still I held on for a time to my own as "the true" explanation of the scale, and for the reason that this style alone contained the three best harmonies of music combined in the most closely related manner possible. To hear this scale in chords, we should strike first the major triad, then its relative minor triad, then the dominant seventh chord, and lastly the major triad again for a close, inverting the chords according to taste. But all these chords should be in natural tune, for if they be tempered according to the uncial scale of 12 equal intervals, all styles of the major scale will be the same, the quincal thirds all rough, and the septals "away off."

I believed in my own discovered rationale of the scale to the exclusion of other styles because it was formed of the three best harmonies possible in music, all arranged in the nearest relation, and because it contained the septal element. The three-triad style did not contain it, neither, on the other hand, did the two-harmony scale contain the *relative* minor triad, except one which was imperfect by 11 nils. So, then, if I at the present time still believed that any scale formed by two or more consolidated harmonies is of any account as a foundation of the science of musical intervals, I would still regard this style of mine as "the true" one; but its finding was chiefly useful to me in "finding" that not the various combinations of harmony, but harmony itself is the solid rock to build on.

As a scale for rapid melody, we cannot usually have the septal element in its purity, for it makes the fourth of the scale very noticeably out of tune with the first of the scale; and for rapid melody also the tempered diatonic scale, which is a part of the uncial scale, is still better than any of the above "true" styles, for thus the bad fifth from re to la, 11 nils out, is made almost exactly perfect, like all the other fifths, that is, only 1 nil short; and the major and minor thirds — well, they have to be 7 and 8 nils out of harmony; but these are quincals, while the fifths and fourths are trials, and an interval of a larger vibrational prime will submit to more abuse than one of a lesser prime, and the less the prime the more prominent the interval.

These three ways of building a "true" major scale are the most noteworthy; and yet we could exercise still farther in this business. For example, take the major triad, do, mi, sol, as before; then plant the minor triad so that its major third would be fa, la, in tune with the do, which would give a re 11 nils lower than in the other three styles; then another minor triad, mi, sol, si. Here is a major scale with a tonic chord, subdominant chord, relative minor (la, do, mi), and other chords, but no dominant chord, either as a triad or a tetrad, which a musical ear could hear without a grimace; for the second of the scale would be 11 nils too low for the dominant chord, and if we wished for a dominant seventh (the septal, 4:7) we could have only its very coarse imitation (9:16), a true double fourth, which is 14 nils too large.

These several scale styles comprise in all ten different tones, instead of seven; and they would appear in one complex scale measured in nils as follows: —

do (93), re (11), re (93), mi (43), fa (14), fa (104), sol (93), la (11), la (93), si (57), do.

But let no one suppose that trying to sing this compound scale as a whole is a useful exercise.

From this showing we can see that to render very simple part-music in tune, even that which does not contain a single "accidental" in the score, it is necessary to use from 8 to 10 tones per octave, instead of 7, as is supposed. In fact the plain tune "America," *alias* "God save the Queen," that is, if rendered in four-part harmony, is partly in this last and fourth style aforesaid, for the minor triad on the second of the scale occurs four times, and the unusually low re is twice in the "tune" itself: do, do, *re*, si, do, re, mi, mi, fa, mi, re, do, *re*, do, si, do. The fa here mentioned is also a part of the same chord, and so is another near the close of the piece, but the low re's belonging with them are in the subordinate parts. The other re's of the tune belong to the dominant chord, they being the usual first step of the scale (8:9) of 104 nils.

Music, then, even very plain and non-chromatic music, is not in one style of the scale alone, but in several styles almost at the same moment. Indeed, the voices of many untutored singers, in part singing, naturally and unwittingly make these *chromatic* — not chromatic — changes quite approximately true. A sense of what the harmonies should be will and should guide singers in this respect, even without a thought or a knowledge of these natural variations of the scale. The uncial scale of the instrument, as properly tuned, makes all the steps either 102 or 51 nils, and does not recognize the (after all rather practical) commas (80:81 and 63:64), measuring 11 and 14 nils. The other commas occurring in this science are not directly related to practical scales.

The minor scale also has several styles, and in fact the instrument and the notation have given it more than one shape. A minor triad for a central or tonic chord, then a minor triad on the true fourth above, and a major triad on the fifth above, make its leading style. This is just the same as the first style of the major scale, except that the third and sixth are each 36 nils lower. The long step between its sixth and seventh is not found in any style of the major scale, but approximates very nearly, not to the true minor third, of 161 nils, but to the little septal third (6:7) of 136 nils, it being 140 nils. On my instrument, the harmon, in which the octave is actually divided into 53 equal parts, this "augmented" second and this small third are identical, and represented by 12 53ds. the minor third (5:6) being 14, the major third (4:5) being 17, and the large third (7:9) being 19 53ds.

There are some curious and not useless things about the several other styles of the minor scale, especially in connection with the septal element; but it would be trespassing too much to bring them in here. The pure harmony 4:5:6:7, however, is chiefly useful in the major mode, though the septal element appears, and grandly too, in connection with the minor mode, but usually in rougher septal harmonies than the above; and it may as well be just mentioned here that if we raise the first tone of this septal tetrad 43 nils (20:21), we shall have, in the purest form possible, that curious creature which is called by musicians the chord of the "diminished seventh," the whole interval, however, being now a pure major sixth, 3:5.

As to "sharps and flats" — a "shower" of them, which I predicted about this time in a former article — well, it is not equinox yet, but only the tenth of September, and too delightful a day to be writing such awful stuff as this.

We have found four kinds of large steps in scales, measuring 118, 104, 93, and 140 nils (this last in the minor scale), and have also seen two much smaller ones ("diatonic semitones"). The last step of any of the styles is unequivocally 57 nils (15:16), unless we count a rather obsolete style of the minor scale, in which it is 93; but the step between the third and fourth in the major mode is sometimes 57 nils and sometimes only 43 (20:21), according to whichever harmonies or scale styles are in use at the moment; that is, mi–fa is sometimes the one and sometimes the other.

I have yet to learn that this larger "semitone" can ever be a chromatic, in principle, in any musical progression; but I have found that the smaller one is *sometimes* a legitimate "sharp." But the "accidentals" are usually, in principle, of three other kinds yet to be mentioned.

In increasing the tonic major triad, for instance, to a double major third, making the minor third into a major — which change might occur in passing "gracefully" into the subdominant chord — we should have the little "sharp" of 36 nils (24:25). (And, by the way, in attempting to give a sort of answer to a correspondent in THE LEADER, some

months ago, concerning "the difference between G♯ and Ab," the former got twice misprinted simply "G," which made nonsense of what otherwise was necessarily a rather imperfect answer to his question. Yet but few papers would print my articles with so few errors as does THE LEADER.) The said subdominant triad is reached by raising both tones of this second major third 57 nils, forming a major third on fa, the do being unchanged. But we don't pass back to the tonic chord by the same road. We can jump back directly, or use an intermediate chord. The la in the subdominant can be lowered by this same little chromatic, 36 nils, which makes the whole triad minor; then lower both tones of the new minor third by 57 nils, and the tonic chord is reached again. Now if we call sol-la in this case G-A, the interval being 93 nils, then the difference between G♯ and Ab, as just shown, is in this instance 21 nils; and this is precisely the same as the difference between three true major thirds and an octave. Yet this interval, which may be roughly called half a "semitone," can never occur directly in any melody or part in music, nor anything very near to it in size.

The other two chromatics in common use measure 47 and 61 nils. The first, which has a very complex ratio, 128:135, occurs in passing temporarily from fa, the true fourth, into a new scale on the dominant, sol. Thus: fa, (47) *fe*, (57) sol. The other and very large one (14:15) of 61 nils is a septal, the two former being quincals. It is the difference between the tonic seventh (8:15, inversion of the larger diatonic "semitone") and the septal (4:7), and is frequently used in passing down from this tonic seventh so as to make a dominant seventh of a new key, namely, on the subdominant. In this case it is a "flat." It occurs also as a "natural," in the very same relation, but by descending from sol instead of from do. In fact, if we descend chromatically from sol to re, we shall have, in the five "semitones" thus formed, four different ones, — and they are, in regular order downwards, 57, 61, 43, 36, and 57 nils in size; that is, the simple harmonies of which these chromatic tones form a part, *make* the semitones thus and so. The 61 is here a "natural," and restores the fa, which is now not the true tonic fourth, but the true dominant seventh (4:7).

But these same "chromatic semitones," and occasionally one or two others unmentioned, occur in various other relations, though these are the most notable cases.

As a summing up, then, of this and the former article, the harmonious chord intervals are so curiously irregular in size, that the steps of the various scales which they make, together with other steps which result from changing tones from one chord to another and also from one scale into another on a different tonic, make quite a good large family, the number being not precisely limited, but not often exceeding twelve, including long steps, short steps, and two very short ones called commas, of only 11 and 14 nils, making just eleven really musical steps treated of in this article. As for various other short intervals which we find in studying this science, they do not need mentioning here, for they do not *directly* concern music.

I have occupied a long *interval of space*, this time, and whether it will reward anyone for reading it, I cannot say; and no one reader will hardly understand the whole of it. And yet there is in my mind a simplicity about this whole field of science which does not probably appear to any reader; and the reason is not merely because I have given so much more attention to it, but chiefly because I learned long ago to see plainly that *the four prime intervals do it all*, not only in the science itself in our minds and on paper, but in reality in music itself, for these four intervals (1:2, 2:3, 4:5, and 4:7) are what we actually sing and tune in voice and instrument, and all melodic steps make themselves by our passing from one harmony into another.

The terms which I have had to use, in this article especially, may not entirely please musicians, and they certainly do not please me; but although sometimes awkward, they are such as I could choose to be at all understood, and maintain anything like definite ideas.

TONE INTERVALS.

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XI.

THIS, the last chapter of the present series but one, will be somewhat of a digression from Tone Intervals; but as the letters which I receive seem to require it, I am just now disposed to make this little departure from the even tenor of my ways.

Nearly all that reaches the public eye on "the temperament of the scale," as well as on musical intervals generally, is not only largely erroneous but largely nonsense. Dictionaries, cyclopedias, books on tuning, and other treatises which refer in any way to these matters, have been heretofore the bane of this wholesome and practical science. I found this out many years ago, when I first became interested in this way. Such published accounts are usually written by ready writers whose information on the subject is second-hand, to say the least; and the real thinkers in this line are not apt to be ready writers — or else not ready to be apt writers.

These articles of mine, therefore, undoubtedly seem somewhat curious to even the best read readers, mainly because I write the truth, or at least have the truth, and write it as well as I can.

With little exception, the letters I have as yet received concerning these articles show that my correspondents are completely at sea. Some gravely inform me that they differ with me, expecting, I suppose, that I shall hasten to inquire, with great concern, in what way they differ with me. If they will write me a letter really worth answering, and kindly enclose a two-cent stamp, both of which most of them fail to do, I will probably find time to answer them with pleasure. Some inform me, too, that they are taking great interest in the articles and learning much from them, but that they don't see nor believe this, that, and the other thing — the self-same fundamental doctrines which I state. Others attack me on "beats," as if my articles were on acoustics; and one has lately sent me a piece of paper with a concise letter on one side and a table of beats on the other — and I must say that the *beets* which I had to-day at the *dinner-table* were much more correct. The numbers in his table, however, show that they had an approximate relation to the truth, but they are mixed, and his intervals are ambiguously expressed.

Now, I will say here, although it is out of the line of my subject — musical *intervals* — that about a year ago I revised and much improved my tables of beats which I made a number of years ago, and that I probably now have, for our common uncial scale ("equal temperament," the octave interval being cut into twelfths or unces), the most accurate table of the beats of the imperfect fifths and fourths ever made. And what seems strange to most persons is, that the beats are by no means equally rapid in the compass of only one octave, saying nothing of a larger compass; although the proper *amount* of variation from each perfect fifth and fourth all through the scale is one nil. The various rapidity of the beats of imperfect intervals, therefore, do not *measure* the amounts of variation any better than do the numbers in the vibration ratios measure the intervals which they form.

The relative rapidity of these beats is calculated with some little care and labor, and a description of the process could hardly be well put into any popular periodical form, a remark which equally applies to the process of finding the true systematic measurements of intervals of tonia from their vibrational ratios, as formerly stated.

Correspondents demand of me how the properly tempered fifth C — G, for instance, should beat; but they cannot tell me — because they do not know — what they mean by C. Most of them will say, "concert pitch," and some suppose that is five hundred and twelve vibrations per second for — well, they don't know whether they mean "middle C" or its octave above, but they really should mean the higher one, sometimes called "fork C," to distinguish it from middle C, which latter would require a fork much larger than the forks in general use. Others would mean French pitch; but they don't know what that is. It is 517.3 for "fork C" — a very ridiculous kind of pitch number, by the way, and it ought to be pitched out of the window. The French pitch is 435 for the A below. The French begin at A, and we begin at C; but it does not matter much where we begin, if we don't get completely *at sea*. If we multiply this last number by six and divide by five (5 : 6 being the vibrational ratio of the minor third), it will give a pitch number for C which is a true minor third above, that being thus 522. But in the uncial scale the minor third is eight nils less than true, which makes it exactly a quarter of an octave, and the French pitch for A, increased by this "tempered" minor third, is ciphered out to be only 517.3, as above. But for the life of me I don't know for what reason the French Academy chose the number 435 (or 870, as they call it, for *one* of our American, English, or German "periodic" vibrations is regarded as *two* by them). I can see good reasons, however, for choosing 512 for a standard pitch-number for C — 256 for middle C, although it would throw away all the present orchestra instruments; for this is so much lower than the present practice and the tuning forks mostly in use that it takes almost or quite an "equal semitone" (unce) to bridge the difference; the high orchestra-pitch being about 540, and 542 is one unce above the 512 pitch.

Now without troubling the reader with very scientifically accurate tables, which involve nice fractions, I will make a few approximate statements about the beats of fifths and fourths of the uncial scale in the middle octave, C — C. No one, however, will ever find them tuned in such very nice exactitude as even the approximate figures here given, unless possibly on a new and carefully-tuned set of reeds (in a cabinet organ, for instance). As to piano-tuning — well, we can usually get an approximately "equal temperament," but not so very approximate a one as to make the beats of fifths and fourths correspond with the far more accurate rule here given. Piano-tuning, therefore, — unlike reed and pipe-tuning, which is nicely done by means of the beats, — is done most advantageously through a knowledge and a familiarity (to the ear) with the properly "sharpened" major thirds, although *their* beats are so very fast that they cannot be counted nor easily perceived; for these properly-sharpened major thirds prove that the four fifths and fourths by which we arrive at them have at least *averaged* the right amount of variation — one nil each; and if the major thirds vary anywhere from five to ten nils sharp (they *should* be just seven), then the "temperament" is a fair one, and if they are anywhere from six to eight nils sharp, that is an uncommonly even temperament for a piano, and the best of tuners could not get such a piano temperament, except by accident, on one piano in ten, and he would be just as apt to get it on a cheaply-made piano as on a high-priced one.

The beats of the fifths and fourths, in our accurate uncial scale, are much slower than those of the major thirds and their inversions, the minor sixths. The beats of these thirds and sixths in the middle octave are too rapid to be counted, being something like twelve times as fast as those of the fifths at

X

THE FRET PROBLEM.

An allusion to the untunefulness of the guitar, in the October Leader, by its entertaining contributor, J. Jay Watson, induces me to offer a few words from my own standpoint, which is a very different one from that of Mr. Watson, who has had so long and so thorough an experience in teaching the guitar, so to speak; while I have but little personal acquaintance with the instrument.

But I actually, once upon a time, — it was thirty-five years ago, — made a guitar; yet I don't know as I could now rightfully claim an acquaintance with the instrument on that ground. A near neighbor, an old cabinet-maker, who was pretty good and patient with boys, although they sometimes dulled his tools, was of invaluable help to me in getting up that amateur guitar. I had never seen more than one or two guitars up to that date, for we did not live in Boston. My desire to possess one, however, must have been very powerful, to enable me to persevere as I did in making that instrument. I rode or walked many miles, not only to see a real *living* guitar, but to procure suitable timber for the construction of one. Well, it was quite a success, and did me credit, and accompanied my brother's violin quite splendidly in one or two keys.

I did not tune it as guitars are usually tuned, by fourths and one major third, and I don't think that the one which I inspected in a neighboring town was tuned in that way. I tuned it harmonically, somewhat as a banjo is tuned, for I knew of no other way.

But the frets! Well, I was sure I could put those in correctly *by ear*, and so did not trouble myself to notice very particularly how the frets were placed on the guitar which I travelled so far to see. I strung and tuned mine first without frets, and then — to make a long story short — got some half dozen frets inserted into the finger board *according to ear*, though I remember of being surprised to find how unsystematic were their distances apart, for at a certain place, for instance, where I wanted the harmonic seventh a good deal (this interval and all others being then, of course, scientifically strangers to me), I had to place one particular fret wonderfully near to the next one.

This will do, for my juvenile experience.

In these my remarks on the rule for placing frets, it is supposed that the strings are evenly made and perfect, though it is probably too true that they are very frequently otherwise.

The real scientific rule for placing frets, and also the rationale of it, are probably known to but a very small number of persons, although all makers of fretted instruments of course have an approximately true rule for the frets, derived from some source; but both their rule and their carefulness in using it may often be defective.

Instruments are fretted on the principle of "equal temperament." This may not be apparent to many readers, but it is just as true as any statement can be. The only perfection, therefore, that can be hoped for in guitar intonation is the perfection of "equal temperament;" that is, the octave cut into twelve equal intervals, which is called, in my articles on intervals, the uncial scale, each "equal semitone" thus formed being called an unce. Fifty-one nils (*nothings* — almost) make one unce, and therefore the octave is 612 nils. Both these divisions are artificial, but Mother Nature does not make the octave and other harmonious intervals divisible into equal parts, and thus we have to make them commensurable, for the purpose of interval measurement at least, and the division by 12 is also a mighty nice *hit* in practical music also, its great redeeming feature being the fact that it allows that ruling interval, the fifth, — and consequently the fourth, — to be within *one nil* of perfection, these two intervals in their perfect state being 358 and 254 nils, and in the uncial scale 357 and 255, or 7 and 5 unces respectively.

As for the thirds and other harmonious intervals, the uncial scale leaves them from 7 to 16 nils astray; but, although this injures harmony of course, these intervals do not quite so imperatively demand to be in tune, either in harmony or melody, as do the fifths and fourths.

The unce, then, in this practical system, represents all the various-sized "semitones" which would occur in the progression of *true* harmonies — and there are some half-a-dozen of them, measuring from about one-seventeenth to about one-tenth of the octave interval.

But now, leaving for a moment these interval measurements, which are one thing, let us look at the vibrational ratios, which are another idea altogether. These are 24 : 25 (the smallest "chromatic," it measuring but 36 nils), 20 : 21, 15 : 16, 14 : 15 — and I will not trouble the reader with others whose ratios are more complicated.

But what is the vibration ratio of the unce, which — in principle — is used on the piano, the organ, and most instruments, the guitar included, in lieu of all these natural "semitones"? It being an artificial interval, its ratio has to be approximated. It is much nearer to 17 : 18 than to any other ratio formed by two whole numbers differing by 1.

[See p. X.]

the same pitches, although the relative amounts of *tonia* variation are only as 7 to 1 (nils).

The beats of any imperfect chord-interval vary according to the pitch on which it is located, and the beats of different intervals also vary on the same pitch. I only state the facts: the reasons must here be dispensed with.

Now, then, after *beating* around the bush, let us have a pretty definite statement of these beats. The beats of all fifths and fourths in the middle octave, C — C, at a medium pitch, say French pitch or a little higher, are about 9 to 18 in ten seconds of time. To be more particular, in our "equally-tempered" or uncial scale, in a more accurate state than it can be actually tuned, all the fifths from C — G up to F — C will beat from about 9 to 12 in ten seconds (thus averaging a little more than one per second), and all the fourths from C — F up to G — C will beat about 12 to 18 times in ten seconds (thus averaging about one and one-half beats per second). The late Mr. Ellis's practical rule of allowing one beat per second for fifths in the middle octave, and a beat and a half for the fourths, is a very good one. It is given in the appendix of his last edition of Helmholtz's "Sensations of Tone" (London, 1885).

It is worth while to remark here that if we use the pitch of 528, a well-known medium pitch used by German and English writers, a trifle higher than French pitch, then the correctly-tempered fifth, D — A, in the middle octave will beat just once per second. It is almost mathematically so, which is a rare occurrence in this whole science, in which only fair approximations are what we have to deal with.

The beats of imperfect unisons and also imperfect octaves are considerably slower than they would be for the same amount of variation of *tonia* in other chords. A variation of one nil, for example, which at the fifth, D —A, would cause one beat per second, would beat only once in several seconds in two D tones varying just one nil in pitch. In fact, such a unison beat would not be generally noticed and would do no harm; it would be a good fair unison in piano-tuning, and not a very bad one on an organ. This illustrates what a mighty small affair a nil is, and that is why I thus named it — it is so near to nothing. Yet there could not possibly be a more lucky hit as a standard of interval measurement, especially for the temperament of the scale.

The beats in the octave below the middle one would be exactly half as frequent respectively, and those in the octave above the middle one are twice as fast, etc.

It is to be observed, also, that a properly-tempered fourth *downward*, from middle C, for instance, would beat just the same as the properly-tempered fifth, middle C — G, these two intervals, whether tempered or perfect, making exactly an octave. But the properly-tempered fourth on this last G, completing the octave, C — C, would beat twice as fast as either the fifth or the fourth aforesaid; and so with all fifths and fourths.

I will grant that it must seem strange to those who are not versed in tone intervals that one nil is the true *measure* of variations which cause such different rapidity of beat, and so with other variations from perfect intervals. It is the fact, however, and it is not a new doctrine either; but it is not at all in the habit of being presented so plainly and correctly.

The fifth, D — A, we will say, beats once per second, and its upward inversion, the fourth, A — D, beats just twice per second, and yet this fourth is increased only just as much as the fifth is decreased, which is one nil, and the two make a perfect octave. The tempered fifth on this latter D would also beat twice per second, and its tempered inversion above, that is, another G — D, would beat four times per second, etc., one nil still being the measure of each variation. But the faster beats at the higher pitch do not offend the ear any more than do the slower beats at the lower pitch.

One thing more, and I will close the "equal temperament" treatment. It is this — that the imperfections of harmony which we submit to in our tempered system are not the very trifling matter of one nil, as might be thought from a superficial view of the subject, but of 7 to 16 nils in the chords formed from the vibrational prime numbers 5 and 7 (quincals and septals), the unce or "equal semitone" measuring 51 and the octave 612 nils. Some assume to write treatises and books on this subject who have only this very superficial view. A book was published a few years ago by a very well-known publisher, in which the writer triumphantly announced the musical scale "completely harmonized," for he had happened to find — by experiment probably — how very small a thing the flattening of each fifth can be, if it is done equally in the whole circle. Not a word was said about the rough thirds which the evenest temperament of the fifths has to leave still very rough.

I have written this chapter partly to show some of my readers how level my head really is in the things I am treating of in these chapters, for I have no doubt that many of my readers have, mentally at least, accused me of being wrong in my statements. Cast the beam out of thy own eye, brother!

Tuners will find in these few general statements about beats a far more correct idea of the subject than I have ever seen in any book on tuning; and I would certainly make a much-needed book for tuners, which would be substantial yet well sifted, if I could see enough money in it to pay a laboring-man a fair price for the service.

[Continued from p. X.]

It is between this and 16:17, but much nearer the former.

In order to show here what I have set out to, I must resort to a still more accurate standard of measuring intervals than that of nils, and that is decimals of an octave to six or more figures, with which I began the series on intervals in the January LEADER. I turn to a large old book of foolscap paper which I have been filling with tables and notes concerning musical intervals for more than a dozen years, and from a table of measured simple ratios from 1:2 up to 100:-101, I extract the following:—

Ratios.	Measurements.
16:17 . . .	.087,463—
17:18 . . .	.082,462+

Neither of these ratios is in itself musical, but they approximate in size to those which are musical; that is, the various "semi-tones," all of whose ratios contain no higher prime number than 7.

Now the unce is .083,333+ of an octave, for the fraction one-twelfth reduced to a decimal gives these figures, and any one can compare them with the other two measurements above; and the nil being .001,634— of an octave (the fraction 1-612th reduced to a decimal), it is found that the ratio 16:17 is about 2½ nils *larger* than the unce, and that 17:18 is a little over *half a nil smaller* than the unce.

The ratio 17:18, then, is far the best approximation, with two whole numbers differing by unity.

But I have made far better approximations of the unce ratio than this, which are in much larger numbers, of course; and they are useful in calculating very approximate pitch-numbers (vibrations per second) in the uncial scale, from any given pitch-number for C or A; and the *nicest* of these approximate ratios which I have made is within *one sixteen-thousandth of a nil* of the mathematical truth!

But for fretting purposes, have we any need to *fret* about a better or any different approximation than that of 17:18? Let us see.

Ratios of vibration are precisely the same as the corresponding ratios of string lengths, only the two numbers forming them are reversed, as I suppose everybody who cares to read this article knows. Two strings, then, as 18:17 in length, and equal in every other respect, would give tones varying in pitch by something almost exactly an unce, falling short of it only a good half-nil, a difference unperceived in practical music.

Now if the first fret of a guitar be placed up one-eighteenth of the sounding length of the string, and the next fret exactly one-eighteenth of the remaind r, and so on, the twelfth fret would give the octave. But how perfect an octave would it give? On the principle we are working, the twelfth fret would fall short of making an octave by between six and seven nils, or about one-eighth of an unce, a difference easily perceived by the ear; and, by another calculation, I have found that, on a guitar whose string has a sounding length of twenty-five inches, the twelfth fret, as above calculated, would lack between one-eleventh and one-twelfth of an inch of reaching the middle of the string, where the octave node is.

But is this shortage to be regretted? No, indeed; for a very little something must be allowed for the tightening of the string in fingering, which of course raises the tone a small trifle, and it may be that this one-eleventh of an inch on the finger-board for the octave is not even enough.

I have taken measurements on a few guitars, and generally find the octave fret *more* than half-way up—sometimes a quarter of an inch more—instead of less.

Now if it be true that this little amount of variation, which results from using *exactly* this *approximate* ratio of the unce, is about the proper amount to allow for the tightening of the string in fingering,—a matter which I have not experience enough with guitars to decide,—then, of course, an exact scale or rule should be carefully made with a pair of dividers, by using strictly this ratio 17:18 *for each size of guitar* made; for if a guitar-maker should use the same scale for a twenty-four and one-half inch string, for instance, as he uses for a twenty-five inch, of course one or the other must be erroneous.

This taking one-eighteenth of the string for fretting is a well-known rule; but I have shown here just what it is and what it will do; and when I first made these close calculations upon it, I was quite surprised at its real excellence, from the fact that the variation of it from the true uncial principle was *in the right direction* to counteract the slight bending of the string in fingering, though the different sizes of the strings will not thus vary exactly alike under the pressure of the finger.

As to tuning the well-fretted guitar, the fourths should be good if tuned by unisons on the frets, that is, averaging one nil sharp; though if some of them are, for instance, perfect, and others even two nils sharp, or even all of them two or three nils sharp, it would be fair; and the major third G-B, thus tuned by a unison, will be, in principle, the rough major third of the uncial scale, seven nils larger than the pure article.

The guitar, then, will have to be called "in tune" when its thirds, say, are like those of almost any tolerably new cabinet (reed) organ. No beautifully perfect third, sixth, or harmonic seventh, can be got from it, except by badly sacrificing some fifth and fourth by re-tuning, or untuning, [illegible]ing.

In a further examination of guitars, since the above was written, I have found that the better ones, those costing $40 and upward, have the octave fret very nicely in the middle of the string, but that in the cheaper makes that fret inclines to be beyond the middle. In both cases, however, the octave tone will be at least five or ten nils too high by the bending of the string in fingering, supposing the string to be even and perfect. The foregoing rule of using strictly the ratio 17:18 will put the octave fret from one-twelfth to one-tenth of an inch *short* of the middle, and thus give not only a better octave, but a better uncial scale throughout.

JAMES PAUL WHITE.

Guitars which are fretted too high, that is, the octave fret receiving be yond the middle of the string (from the nut to the bridge) are in fact better to be so, rather than vice versa. The writer [illegible] (Guitars) has this advantage [illegible] length of the string longer. The bend [illegible] of the string & to 4 [illegible] while bending [illegible] twice the distance from the nut to the fret.

[For The Leader.]

RHYTHM.

When o'er primeval, vast, chaotic space,
God flashed the day-break of creation's morn,
Disordered elements to law gave place,
And from discordant force was rhythm born.
Then in grand diapason burst of joy
The new-born morning stars together sang,
A unison whose mighty strains deploy
And through the ages since like joy-bells rang.

In His own image God created man
And in his nostrils breathed His living breath;
So with each pulsing throb is told His plan
Which marks life's border lines 'twixt birth and death.
Each throbbing heart-beat speeds the course of time
With metronomic fervor, day by day,
From youth's warm morn to age's evening rime, —
So life's brief rhythm runs its course alway.

The tiny shoots of green that pierce the shroud
Which drapes the earth in winter's fleecy snows, —
The violet, with modest grace endowed, —
Trailing arbutus and the queenly rose.
The waving grass that bends beneath the breeze,
Then falls before the sturdy farmer's blade, —
The ripened grain that waves in golden seas,
Or waiting, stands in serried sheaves arrayed.

The whirling planets march in astral beat
And fill the measure of the passing year,
While myriad stars with scintillating feet
Dance sparkling cadence in the azure clear.
The gentle moon enthroned in regal state,
Moves on serenely o'er the heavenly arch,
While kingly sun, her grand, majestic mate
In royal strides leads the eternal march.

The mighty ocean, stretching far and wide,
From polar ice to tropic's vernal sun,
Bearing along its ever restless tide
Winged messengers which make the nations one, —
Each tiny wave that ripples on the shore,
When gentle zephyrs lull the deep to rest,
Each giant billow, in the stormy roar,
To rhythmic measure times its foamy crest.

Thus ev'ry phase of time this law controls,
Each act and principle its pulsion owns;
Tides ebb and flow; each mighty planet rolls
In grand precision through the belted zones.
Thus ticks the patient clock, thus beats the heart,
So throb the pulses as life's current swells.
Thus music's sweetest cadence links with Art,
So ring the birth, so toll the funeral bells.

FRANK N. SCOTT.

TONE INTERVALS.

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Errata. — There have been but few serious errors of print, for such a subject; but in the lengthy October article (the proof of which I could not see as usual) *commatic* — an adjective from *comma* — was printed *courmatic*, and *tuned* — referring to the accordion scale — was printed *termed*. The word *commatic*, however, I don't think much of, for if we say *commatic changes*, it sounds too much like *chromatic changes;* but this is the very reason why, in that particular connection (in the jingle of words), I used that word instead of *enharmonic;* and, by the way, this word itself was printed *in*harmonic in the second column of the January article, and further down, the second *l* was omitted in *Helmholtz*. In the June number, *tempered* scale was *temporal* in one place; and in July, the quincal was "4 : 7" in one place, instead of 4 : 5. But these errors, and a few others of less consequence, were my own fault, for they escaped my eye in the proof, that of the October number excepted.

XII.

In this chapter — which I am afraid will bear some resemblance to half a pair of shears to those who have not read former ones — the first, and perhaps about the only thing I can now take up, is a little more about the uncial scale, the scale of equal "semitones" or "equal temperament," though this latter expression is not as useful in our day as it might have been, say, a hundred or even fifty years ago, when there was considerable difference of opinion in Europe with reference to the different systems of tuning or "tempering the scale." The unequal systems were used in organs and other instruments for hundreds of years before our century and even into the middle of it; and I think there are corners of Europe, where unequal temperament is still in vogue.

Of course I mean a *system* or *systems* of unequal temperament. As for the *accidental* inequality of temperament in which pianos especially are often found, that is counted out.

The even uncial scale does indeed damage the musical beauty of all the thirds and all other chords formed by the vibrational prime numbers 5 and 7; and those uneven systems of dividing the octave in the past did give some of the thirds perfectly, and very nearly so, but gave others far worse than in the uncial scale, and therefore made them unusable pretty much, and gave many of the fifths also sensibly discordant, both flat and sharp; and with only twelve tones per octave, and with a free use of every one of the twelve keys to play in and modulate into, the uncial scale is the only thing which can at all "fill the bill" for most modern music. We thus lose something, however — yes, we do indeed; for we hear the rough thirds *constantly* in every harmony, and have thus small chance to cultivate the acquaintance of the true thirds, which those who lived in the days of the unequal "temperers" did have. It is no wonder, therefore, that people who live in this *uncial* age of music have forgotten (?) — because they never knew — what little used to be known in this line to musicians a hundred or more years ago. This kind of musical learning has, unfortunately, passed out

of fashion, and the universality of the uncial scale — whose immense value I appreciate as well as any one can — has driven this mathematical knowledge of music, what little there was of it, out of musical society. The scale of equal "semitones" for that is what it is, whether the musical mind is always aware of it or not — is supposed to be "right," "in tune," etc., by which it seems to be well nigh meant that God made it so, and gave it to Moses on Mt. Sinai! while the fact is, that while "equal temperament" was strongly favored by both Sebastian Bach (d. 1750) and his son Emanuel (d. 1788), for clavichords at least, and thus made headway in central Europe in the last century, it was not a firmly established thing in England for organs till about the middle of our own century, and not probably earlier in the United States.

Some of my readers can doubtless remember — for I can myself, and I am not yet a hundred years old! — how charmingly *some* of the chords used to sound in the little *lap* melodeons of forty and fifty years ago. The major chords of C, of G, of F, and perhaps more, had perfect or very nearly perfect thirds. I did not know, however, in those early years, what I afterwards discovered — and on one or two of the very instruments — that numerous other triads were necessarily yery unmusical. In the chord of A*b* major, for instance, the major third was "way up," — it must have been at least 14 nils sharp, which is twice as much as the "equal" major third is.

Those instruments, therefore, thus tuned, were very limited in their sweetly musical resources for modulation and for keys to play in. But time will never efface the impression which was made upon our tender little souls by the few naturally tuned chords; and when years afterwards, while growing into manhood, I formed the acquaintance of the larger and improved melodeons, having from four to six octaves and placed on legs, I used to wonder why it was that their harmony was not so truly musical as that of the little "teetering" things which I first heard in childhood. I supposed that all melodeons were "in tune" of course — of course I would not dispute the *tuning* of such a *heavenly* instrument as a *melodeon!* A violin — no, I mean a fiddle; we boys never said *violin* except perhaps when we had company — I could probably tune nicely then, it being in fifths, but thirds I had no experience in tuning, except as I fingered them according to a pretty nice juvenile ear. But I distinctly remember a puzzled feeling I had from listening to the thirds on those later melodeons made about 1850-60, which I now know were uncial, such as are now tuned, being natural major thirds increased by 7 nils, so as to make them just one-third of the octave interval, the unce being 51 nils, and twelve unces or the octave, therefore, 612.

But as I said nothing about it to any one, and would have stood a slim chance of enlightenment if I had mentioned it, it was more than a dozen years after the discovery of the unsatisfactory thirds before I began to know that the truly harmonious major third was one thing, and that the major third of our equal "semitone" or uncial scale was a different thing.

I ought to relate in all honesty, in this connection, the success (?) of my first attempt to tune any kind of an instrument having a keyboard; and it was a melodeon, one of those later ones, on legs. This was years before I studied up tuning at all, but I knew how to raise or lower the pitch of a reed by scraping or filing; — and hadn't I an acute "ear," and that, too, all the way from childhood? What, then, was the matter that I shouldn't tune that melodeon? At it I went, and it was to be a nice job too. Thirds, fifths, etc., were tuned to perfection no doubt, and how beautifully they sounded too, just like that first little "teeter" melodeon of yore! I was getting on splendidly — for a few chords — till I got to where chord had to *meet* chord. Gracious! what did it mean? Nothing would come out in harmony, nor anything decently near it. The E which I had tuned as a harmonious third for the C triad wouldn't agree at all with the E which I got by tuning perfect fifths and fourths from this same C; for their difference, the comma of 11 nils, I knew not of. And, worse yet, when I piled *two* perfect major thirds on C, I got something for the black key between G and A which was a curiosity certainly, when sounded with the upper C, which would be supposed to be its major third; but this C is 21 nils, or towards half a "semitone," too high for that, and it is the perfect octave for the lower C, and must be so. And this extra 21 nils (the diesis) is divided equally in the uncial scale between the three major thirds, which, on the keyboard, must make a perfect octave: — and as to that melodeon and its tuner (?) — well, we will let the matter drop; but I will just say that, had I been a believer in witches, my faith in witchcraft would have been confirmed by that youthful experience; and in after years when I had become a tuner, I would sometimes come across a melodeon, and also a reed organ, which had evidently been tampered with by some other fool who supposed that a good "ear" alone, unaided by knowledge, can tune an instrument which has twelve keys per octave. And yet this uncial scale can be tuned and actually is tuned, in a reed instrument at any rate.

and even by one who has no true conception of this whole subject at all, provided that he simply knows how to make the fifths and fourths *beat* about right (as shown in the last number), *and will let the thirds alone*, which, of course, will all come out rough, but equal; that is, the major thirds will thus be one-third of the octave interval, and the minor thirds one-quarter of it. I refer particularly to reed and pipe organs, which can thus be tuned quite nicely in the uncial scale by means of the proper beats in the slightly imperfect fifths and fourths; but in pianos, whose very nature, being stringed instruments, forbids their being tuned always so evenly, it is necessary to make large use of the properly sharpened major thirds as proofs; and this, I think, is done most advantageously — in pianos — by the cultivated sense of tune, without much aid from beats; that is, the tuner of pianos, in order to set a tolerably "even temperament," should readily recognize by ear a pure major third, and also thus determine pretty accurately *how much* the properly tempered or uncial third should overrun it, that amount being 7 nils.

In the series closing with this number, I have probably repeated certain things too much for some readers, especially those who have read from the beginning; but otherwise each article by itself would have often been meaningless, in too many places, to those who could not see the preceding ones. But I have not attempted a well arranged treatise in any of them, and have only sought to break the ice a little; or, if that figure won't do, I have merely *introduced* the mathematical phase of music, and in a quite different manner from what others have done; but there is, in my own toniametrical — an easier word to pronounce than *trigonometrical* — storehouse, much more and far more which I am secretly threatening to publish at some time, together with these fundamental doctrines put into a somewhat different form. But all who care a fig for these present articles had better preserve them, as I egotized away back towards the beginning; for they are rather original, and will not be republished; and it must be remembered that nearly all papers, even those as valuable as THE LEADER (though that is now dropping the newspaper form), are soon destroyed.

Some readers, who happen to know something about the matter, may wonder why I have not given some considerable account and description of my musical instrument, the harmon, in these articles. I really somehow couldn't get to it. I chose to continue writing on the principles of Intervals, and leave that little undertaking to take care of itself some other time. I devoted much time and pains, some years ago, to the inventing and making of several of these instruments; but have little taste for mere curiosities, and have little delight in merely announcing to others that I have made a reed instrument with 53 or more keys per octave. I have much to say about the harmon, but not now.

Various other points of practical interest will have to be put by, this being the closing chapter; but to those who are at a loss to know how the number of vibrations per second of time (called the pitch number) for any particular tone, or, by combining two of them, for any particular interval, is ascertained, the answer is, that an acoustical machine called a *siren* (sometimes spelled *sirene*) is the usual means of illustrating and proving these facts. They are not very common, but are found in a few institutions of learning, where natural philosophy, or physics, is extensively taught. Descriptions of various kinds of sirens are found in Helmholtz's Sensations of Tone, English editions of 1875 and 1885, London. The machine, in its simplest form, consists of a revolving wheel of thin metal, like a circular saw without teeth (which might be made, for an experiment, of stout card board), in which is a circle of round holes, say 12 in number; and then inside of that another circle of, say, 8 holes, the *ratio* between these two numbers thus being 2 : 3, which is the vibrational ratio of the true fifth. If two blasts of air be blown directly against these two circles of holes from a bellows, through two fixed pipes, while the disc is revolving swiftly, two weak tones can be heard tuned exactly to a fifth, the air blowing against the inner circle of holes giving the lower tone. And it makes no difference with the *interval* how fast or slow the wheel revolves, provided it turns fast enough to produce an audible tone at all: and it must revolve more than once per second to give even the lowest organ tones (not in *quality* of course, nor in volume, but only in pitch); the lowest audible tones being in the neighborhood of 20 vibrations per second.

and also syren

The siren can, moreover, determine the absolute pitch number of a single tone, as well as it can give natural intervals without reference to absolute pitch, by a calculation on the number of times the wheel revolves per second, and then by multiplying that by the number of holes in the circle; and the siren can also be so made as to give any natural tone interval or intervals whatever; and if the revolving disc were made with five circles of holes, and, beginning with the innermost circle, the holes were in number 4, 5, 6, 7 and 8, or 8, 10, 12, 14 and 16, all the chord and interval elements of music would be heard together in one perfectly harmonious concord; and it would be the full chord of the harmonic seventh, with the octave added, vibrating as 4 : 5 : 6 : 7 : 8, which I have treated of in these articles, both as to the vibrational ratios and the interval measurements, and have attempted to state also how all really musical intervals are formed from having no larger prime numbers than are herein contained, that is, the number 7.

And now, as a final benediction, I might answer the question, *Cui bono?* but it would make too long a benediction. What is the *good* of all this stuff about Tone Intervals? for I presume some have, mentally at least, asked the question. This phase of music, which for twelve months I have been here talking about in a very irregular manner, is the very *sine qua non* of musical science; and this statement alone — if indeed true — should clearly enough show the usefulness of such knowledge. I am myself just as dissatisfied with anything I have as yet written about it as any one can be. Not that I at all doubt the truth of what I have attempted to bring out, but that I always feel that the medium of communication is imperfect and sometimes misleading at best. My readers must forgive me when I say that I cannot possibly believe that ours is a great age for *musical* culture. With all the musical activity of our time, I realize — and of course many others do, but perhaps not with the same views — a tremendous waste of the real article, music.

I am no "perfectionist," I only claim to have common-sense — added to a diligently acquired knowledge of and familiarity with the phase of music which has thus become my specialty. I found this field of science, years ago, neglected and abused. I immediately began to realize its usefulness; and the work of developing it, in connection with constant practical work as a tuner, suited my taste and inclination; and yet I have been always more or less at a standstill as to the means of communicating the science, our musical terminology being so poor for the purpose, and the very first principles of the subject being so largely unknown and unappreciated; and this series of articles is more than all my other published writing on the subject, as yet, put together.

An acquaintance with this branch of learning — supposing we had text-books for it, as we have for other branches — would enable one to realize where the true merit of our uncial scale is, and would prevent him, on the other hand, from trying to extract from it what is impossible. It would prevent vocal teachers from trying to compel their pupils to sing exactly to it, even when best tuned, which is unnatural and injurious to the voice, however good that scale is in melodic runs and flourishes; but all *accented* tones of the voice are never very musical when following strictly the impure *thirds* of this equal piano or organ scale; and, worse than all, the attempt to do so tends to destroy the "ear," so to speak, with reference to the most delicate and charming of tone intervals, the quincals and the septals; neither could a Paganini nor an Ole Bull have ever charmed an audience, on an instrument of four strings or less, if they had followed, especially in their accented tones, this scale of "equal temperament."

Now, then, good-by for the present to the natural classification of Tone Intervals, their measurements, and their comparison with those of the present practical uncial scale, which knows no intervals except such as are twelfths of the octave interval; and *tonia*, which I certainly did not *discover*, for everybody tacitly assumes it to exist, but which I seem to be the first to duly recognize, honor, and name, as a species of mathematical quantity, will not probably forever remain in obscurity; but this series about Tone Intervals, or Toniometry, is now ended.

JAMES PAUL WHITE.

END OF THE SERIES.

[A new and interesting series of articles, by Mr. White, will be commenced at an early day.]

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155